Foreword

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To my knowledge, this compilation is the first book entirely devoted to the phase singularities of light, marking the coming-of-age of the subject. Therefore it is appropriate here to recall the circumstances in which John Nye and I wrote our original paper[1] on wave dislocations in 1974, and also to set optical vortices in a more general context.

In those days, Nye was studying the flow of ice sheets, for example in Antarctica. Following the discovery in the 1960s that radio waves with wavelengths of about 5m can penetrate polar ice, radio echo sounding had been systematically employed to probe the topography of the rock beneath the ice, and thereby make the first maps of Antarctica - a substantial fraction of the world's land. In this technique, pulses (a few waves long) are sent into the ice, and the time delay before detection of the beginning of the echo is measured back at the position at the transmitter.

Nye pointed out to me that this ignores the diffraction information in the long and disorderly tail of the reflected wave. I began to investigate the statistics of the diffracted radiation. Nye was more practical, and scaled the problem to laboratory dimensions by replacing the radio waves by 1mm wavelength ultrasound, transmitted by a transducer that also acted as a receiver, and the rough rock surface by a sheet of crinkled metal kitchen foil. With this 'analogue computer', the fine details of the echo tail could be studied – including its phase – for different positions of the transducer. He noticed a peculiar phenomenon: occasionally, movement of the transducer caused two maxima in the echo to separate and give birth to a new maximum between them. (Of course, the time-reverse sometimes occurred too.)

Thinking four-dimensionally, he realized that such events corresponded to the passage through the transducer of a *line singularity* in the pattern of wave crests. This line singularity was analogous to dislocations in the pattern of atomic planes in crystals.

Thinking together about the implications of Nye's experiment, we came to understand that:

- dislocations are generic properties of wavefields;
- they are properly regarded as singularities of *phase* $\chi(\mathbf{r},t)$, defined by suitably representing the physical wave as a complex field $\psi(\mathbf{r},t)=\rho(\mathbf{r},t)\exp\{i\chi(\mathbf{r},t)\}$ in spacetime;
- wave dislocations are *line zeros* of ψ; that is of the modulus ρ; with (integer) strength $S=\oint\nabla\chi\cdot d\mathbf{r}/2\pi$;
- the analogy with crystals is far-reaching: wave dislocations can be of edge, screw, and mixed edge-screw type, can move relative to their host wavefronts by climb and glide, can form arrays like grain boundaries in crystals, and the concept of Burgers' vector applies;
- wave dislocations are the result of *destructive interference*, and so generalize the familiar (and unstable) dark fringes of optical interferometry;
- around a dislocation, the phase gradient lines (along which energy flows) form vortices;
- dislocations can exist not only as moving lines in wave-trains but as fixed lines in monochromatic waves, and as points in waves in two space dimensions;
- exact explicit solutions of wave equations can be found, corresponding to dislocations of many different kinds, moving and interacting in many different ways.

A referee of our paper recommended rejection, because the central idea was 'trivial'. We were able to convince another referee, and the editor, that simplicity was a virtue, and the paper was published.

Many particular examples of phase singularities had appeared in the literature before our general analysis. Every physicist is familiar with the Dirac string emerging from a magnetic monopole in the solution of the Schrödinger equation for an electron in the presence of this hypothetical object, and with quantized vortices in superfluids and quantized flux lines in superconductors. However, phase singularities are much older. The earliest known to us is described in two astonishing papers[2, 3] by Whewell in the 1830s. He was studying the tides, regarded as a water wave of 12 hour period. Following Young, he considered the wavefronts of the tide, defined as lines connecting points where the tide is high at particular times; he called them 'cotidal lines'. By analyzing observations of the times of high tide at different places, he inferred the presence of singularities in the pattern of cotidal lines, that is places where the tide is high at all time, so that the tide height is zero. He called these places 'amphidromic points' (from the Greek, because the cotidal lines ran round the points like the spokes of a wheel); of course, they are wave dislocation points in this two-dimensional wave. In optics, phase singularities had appeared (without being emphasised) in isolated computations of several diffracted wavefields; one is reproduced in the famous treatise of Born and Wolf[4]. There are several equivalent terms for these physical objects: phase singularities, wave dislocations, optical vortices, and (in two space dimensions only) topological charges.

It is not surprising that the general concept of wave dislocations emerged in the Bristol physics department, where there was an implicit culture of line and phase singularities, including Charles Frank's analyses of crystal dislocations and liquid-crystal disclinations, and the Aharonov-Bohm effect; later it led to geometric phases. I have explored this local history elsewhere5.

Wave dislocations belong to the larger class of wave singularities. Historically and conceptually, the first singularities are the caustics of geometrical optics; these are envelopes of families of rays. In recent decades, this ancient subject has been rejuvenated by the discovery by Thom and Arnold that the forms of caustics that are stable under perturbation (and so occur naturally) can be classified as mathematical catastrophes. This has led to a new branch of optics[6], in which the diffraction patterns that smooth away caustic singularities are classified too. The first of these 'diffraction catastrophes' was Airy's function[7] describing the rainbow caustic.

The evolution from rays to waves brought a new concept and a new physical field, namely phase, whose singularities are the subject of this book. Note that Airy's pioneering study of waves near caustics and Whewell's discovery of phase singularities were both made in the 1830s.

The Huygens-Fresnel discovery that light is not a scalar disturbance but a (transverse) vector wave led to a further evolution, and the new concept of *polarization*. Polarization has its singularities as well. The first of these are the conical degeneracies in the optics of biaxial crystals, discovered by Hamilton[8], corresponding to propagation directions where the two orthogonal polarizations coalesce. This 'conical refraction' was also discovered in the 1830s – truly a miraculous decade, in which all three types of wave singularities made their appearance. Nye[9, 10] has explored several kinds of polarization singularity, including a natural generalization of dislocations to vector waves.

Nowadays, the study of singularities has become a central preoccupation of physicists (cf. black holes, and critical phenomena in statistical mechanics), so it is not surprising to see it emerging in optics too. A comprehensive introduction to all the wave singularities can be found in the new book by Nye[11].

For the particular case of phase singularities in light, much of the revived interest has come from the availability of lasers, enabling optical fields to be created in great variety and manipulated easily. Many groups of researchers are involved in this activity throughout the world, especially in the countries of the former Soviet Union – perhaps because this kind of physics requires imagination and ingenuity but not expensive equipment. I had the pleasure of witnessing this new area of 'singular optics' at a conference in Crimea in 1997. As is often the case, experimental activity refocuses the attention of theorists, and I anticipate many unanticipated new ideas.

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