FINDINGS ON LIGHT

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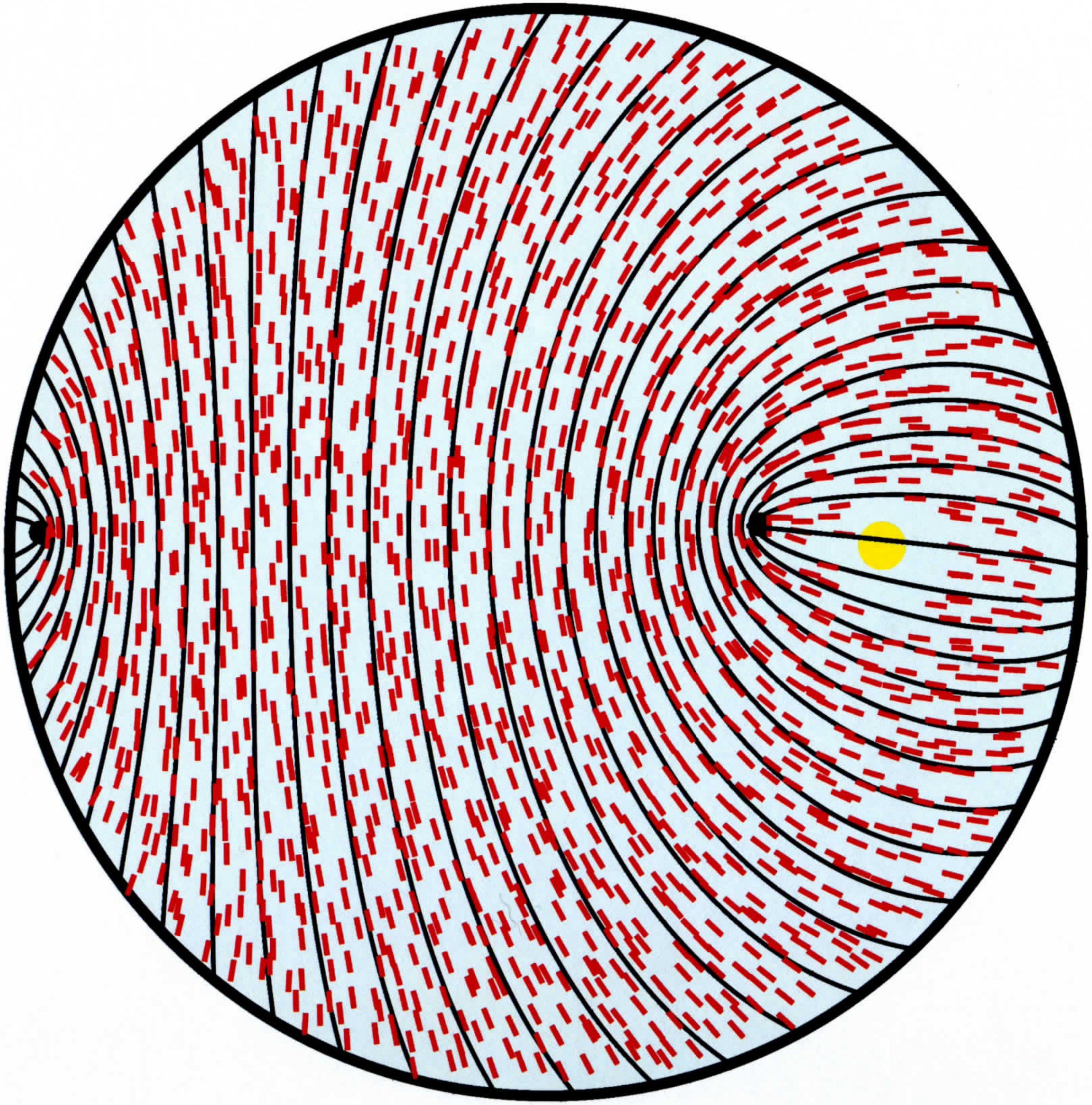
PARS Foundation
Lars Müller Publishers 2016



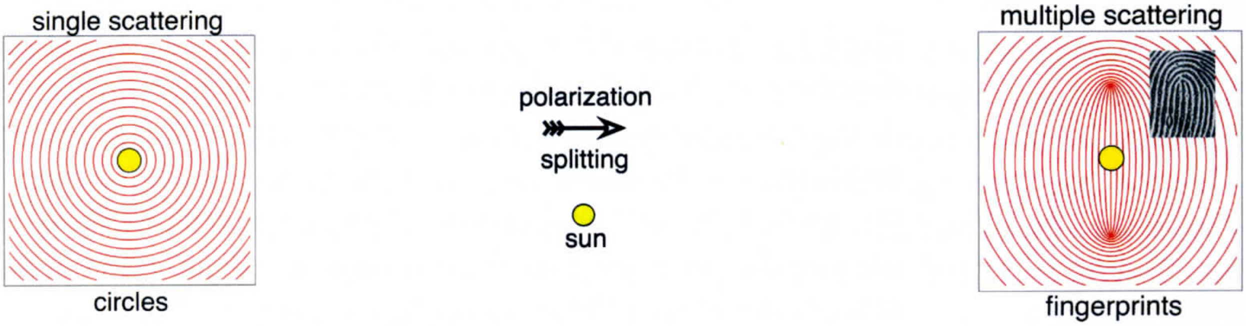
Michael Berry, physicist

THE ELLIPTIC INTEGRAL IN THE SKY

POLARIZATION FINGERPRINTS PATTERNS SKY SCATTERING
YELLOW SUN NAVIGATION

The light of the day sky is sunshine scattered by air. It is blue because the blue waves in the spectrum from the sun are shorter than the red waves, so an air molecule appears bigger to arriving blue waves and scatters them more strongly. But there is more to daylight than the brightness and colour that we see. There is a hidden property – *polarization* – that we can scarcely perceive (though some creatures – for example bees – can). Polarization is the sideways vibration of the electric and magnetic fields comprising light. In the sky is a secret pattern of polarization, built from four fingerprint structures.

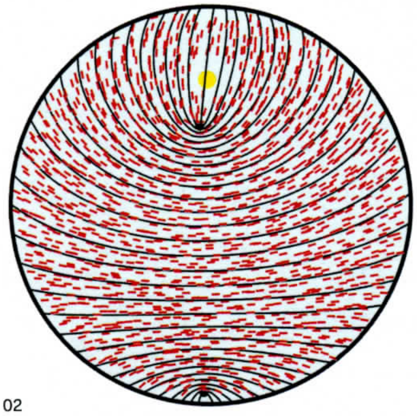
The starting-point for understanding this is that light arriving from the sun is a random assembly of transverse vibrations in all directions – not polarized at all. But the act of scattering induces polarization. Each molecule is a tiny aerial, set into vibration by incoming light and re-radiating it in all directions. Different incoming polarizations are scattered differently, in a way that appears strongest at right angles to the sun, and weakest in the direction of the sun and opposite to the sun (the "anti-sun", visible for a while before sunrise and after sunset). This can be seen by looking at the sky through a polarizing sheet, such as a lens from polaroid sunglasses, and rotating it. The brightness varies, more powerfully at 90° from the sun and hardly at all near the sun and the anti-sun.



01

In this theory, based on the sunlight being scattered once, the polarization pattern of daylight would be structured around the two unpolarized antipodal points in the sky: the sun and the anti-sun. But the reality is different. Already in the early 19th century – and long before the scattering of polarized light was understood – three unpolarized points were observed in the sky: above and below the sun, and above the anti-sun. A fourth was predicted, below the anti-sun, and it was observed in 2002, from a balloon flying after sunset. The reason for four unpolarized points, rather than two, is that light is scattered more than once. This multiple scattering is weak, but its effect is to split each of the unpolarized points in the one-scattering theory into two.

The four unpolarized points, and the pattern of polarization across the sky, emerged from elaborate multiple-scattering calculations. But these features can be understood in a simpler way, based on considering the geometric nature of the unpolarized points. Around each unpolarized sky point in the one-scattering theory, the direction of polarization (for example of the electric field of the light) turns once, and in the same sense. In mathematical terminology, such an unpolarized point is a *singularity with index +1*. When multiple scattering splits it into two, the resulting unpolarized points are singularities of index +½: around each of them, the polarization makes a half-turn. This gives rise to a local pattern like the ridges on a fingerprint, and the reason is similar: polarization is not a direction with an arrow (that is, not a vector), but rather an oriented line – direction without a sense – which looks the same after a half-turn as well as a full turn. That is why index +½ is natural for polarization. [01]



02

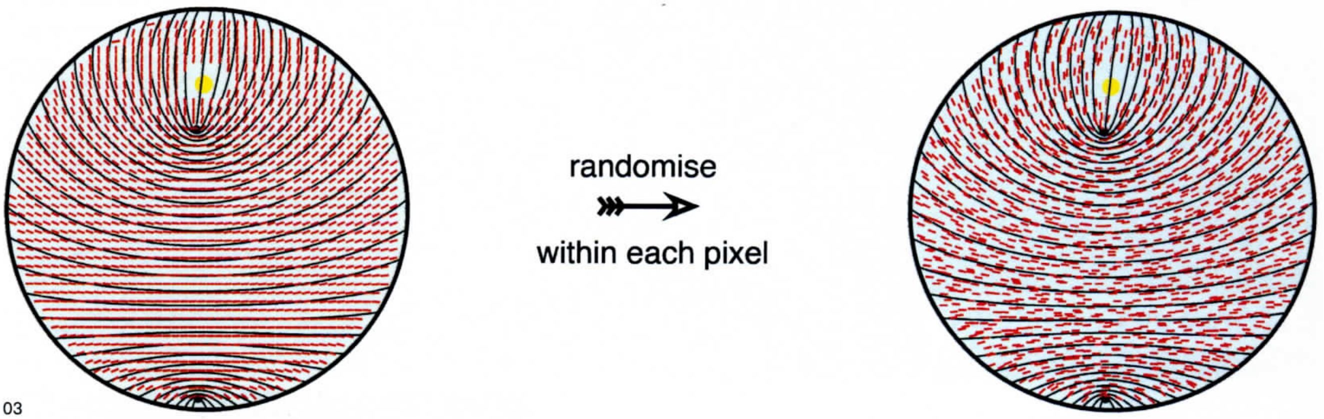
The full polarization pattern lives on the full sky sphere of directions seen from Earth, and is based on the four fingerprints. Of course we don't see the full sphere: at any one time, our sky is the hemisphere above us, with just two of the fingerprints. On the full sphere, the pattern can be built by interpolating between the two antipodal fingerprint pairs of unpolarized points. The simplest way of doing this was devised in 2004 in collaboration with Mark Dennis, and agrees accurately with measurements of the sky by Raymond Lee at Annapolis, Maryland on 3 November 2003. [02] The yellow sun was 33.7° above the horizon. The measured polarization directions are the little red line segments. The pattern predicted from theory is represented by the black contour lines in the sky: at each sky point, the polarization direction is parallel to the contour passing through that point. Contour lines of what? Of a function built from *elliptic integrals*. As the name implies, these were originally devised in the 19th century to calculate the length of the perimeters of ellipses. But they have many other applications, for example to the bending of elastic wires, light rays refracted by sound waves, the statistical mechanics of particles on lattices ... Their appearance in the sky was unexpected.

I wish this nice picture was the one we published. It wasn't. Instead, in the version we published, the observed polarizations – the little red line segments – were plotted as Raymond Lee measured them: on a rectangular grid in the sky. But the grid tends to deceive the eye: we see the lines of the grid of line segments, rather than the orientations of the individual segments. The agreement between theory and experiment is much less clear. An obvious fix is to randomize the positions of the line segments [03] within each pixel, while, of course, preserving their orientations, namely the polarization directions. This rendering by Mark Dennis eliminates the distracting effect of the grid, while preserving the data to the accuracy with which it was measured.

$$\text{contours:}\quad \mathrm{Im}\,F\left[\arcsin\left[\mathrm{i}\,\frac{x+\mathrm{i}\,(y+y_S)}{A\,(1+\mathrm{i}y_S\,(x+\mathrm{i}\,y))}\right],\frac{1}{A^4}\right]=\text{constant}$$

F = elliptic integral of the first kind, x,y = coordinates in the sky,
A = fingerprint splitting, y_S = sun elevation

We cannot see this pattern directly, but with polarizing crystals we can detect the polarization direction at each point in the sky. And although the basic theory was devised by considering the clear-blue sky, it is remarkable that the pattern of polarization directions persists even in cloudy or foggy skies, although the strength of the polarization is weakened. This has reinforced the speculation that the Vikings might have used polarization to determine the position of the sun, aiding navigation westward from Norway via Iceland to what is now Canada. I share the scepticism of historians about this conjecture, because the only historical evidence that might support it is an enigmatic mention of a "sunstone". Nevertheless, it is interesting to explore "what if" history, and investigate ways in which polarization could conceivably have been used, for example using Iceland spar, polarizing crystals that can be found naturally.



03

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1 M.V. Berry, M.R. Dennis, R.L.J. Lee (2004), "Polarization singularities in the clear sky", *New J of Phys,* 6:162

2 G. Horvath, B. Bernath, B. Suhai, A. Barta, (2002), "First observation of the fourth neutral polarization point in the atmosphere", *J Opt Soc Amer A*, 19:2085–2099

3 R. Hegedus, S. Akesson, R. Wehner, G. Horvath (2007), "Could Vikings have navigated under foggy and cloudy conditions by skylight polarization? On the atmospheric optical prerequisites of polarimetric Viking navigation under foggy and cloudy skies", *Proc R Soc A*, 463:1081–1095

4 L.K. Karlsen (2003), *Secrets of the Viking Navigators*, One Earth Press, Seattle