

(translation: *Russian Mathematical Surveys*) **30**, 1–75 (1975); Berry, M. V. 'Waves and Thom's Theorem' *Adv. in Phys.* **25**, 1–26 (1976); Duistermaat, J. J. 'Oscillatory integrals, Lagrange immersions and unfolding singularities' *Comm Pure App Math* **27**, 207–281 (1974)). The classification describes caustics that are 'structurally stable', that is those whose forms survive perturbation. This makes catastrophe theory particularly suited to the optics of nature rather than artefacts such as microscopes and telescopes whose focussing is dominated by cylindrical symmetry.

We have made progress in understanding the optics of irregular water droplet 'lenses' (Berry, M. V. 'Waves and Thom's Theorem' *Adv. in Phys.* **25**, 1–26 (1976); Nye, J. F. 'Optical caustics in the near field from liquid drops' (submitted to *Proc. Roy. Soc.*), the fine structure of swimming pool caustics (Berry, M. V. & Nye, J. F. 'Fine structure in caustic junctions' *Nature* **267**, 34–6 (1976)), atom scattering by crystal surfaces (Berry, M. V. 'Cusped rainbows and incoherence effects in the rippling-mirror model for particles scattering from surfaces'. *J. Phys. A* **8**, 566–84 (1975)) and the statistics of twinkling starlight (Berry, M. V. 'Focusing and twinkling: critical exponents from catastrophes in non-Gaussian random short waves' (*J. Phys. A*, in press)). This last application (which has proved peculiarly resistant to more conventional forms of analysis) makes essential use of the enormous extension of Thom's classification being developed by Arnol'd (Arnol'd, V. I. 'Critical points of smooth functions and their normal forms' *Uspekhi Mat Nauk* (translation: *Russian Mathematical Surveys*) **30**, 1–75 (1975)) in the Soviet Union.

The other area is fluid mechanics, where the elliptic umbilic suggested the design of the 'sixroll mill' (Berry, M. V. & Mackley, M. R. 'The sixroll mill: unfolding an unstable persistently extensional flow'. *Phil. Trans. Roy. Soc. (London)* **287**, 1–16 (1977)), a device for studying the effects of dissolved long-chain molecules on the flow of Newtonian fluid. The mill produces a sequence of flows with fully describable instabilities, and addition of polymer is dramatically revealed by changes in the topology of the pattern of streamlines. This specialised application has now been generalised (Thorndike, A. S., Cooley, C. R. and Nye, J. F. 'The structure and evolution of vector fields and other flow fields' (submitted to *J. Phys. A*)) into a comprehensive theory of flow patterns, which has already given insight into the structure of the geostrophic wind and the move-

SIR,—It would be a pity if the strong attack by Zahler and Sussman on some biological and sociological models based on catastrophe theory, (27 October, page 759) were to mislead readers into thinking that such new and beautiful mathematics has no useful application in any science. The fact is that in this laboratory catastrophe theory is being employed in the development of new concepts, in the explanation and prediction of phenomena, and in the design of experiments, in two areas of physics.

The first is short wave optics (and quantum mechanics) where Thom's theory classifies the forms of focal surfaces (caustics) and makes it possible to give a precise description of the finest detail in the associated diffraction patterns (Arnol'd, V. I. 'Critical points of smooth functions and their normal forms' *Uspekhi Mat Nauk*

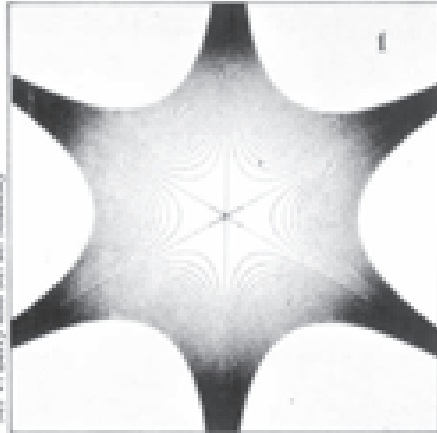
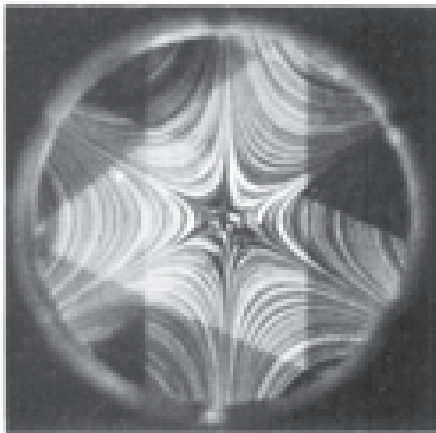
ment of ice in the Arctic ocean.

These are genuine applications of catastrophe theory; they have led to advances in our understanding of the physical systems concerned. It is important to distinguish them from illustrations of the theory, where the mathematics is employed correctly (that is to systems satisfying its axioms) but in more sophisticated derivations of results already known; elastic buckling and the mean field

theory of phase transitions fall into this category. The applications should also be distinguished from what I shall call invocations of the theory, where it is employed because of the suggestiveness of its images in the hope that its axioms might eventually be shown to apply; perhaps it is towards this area that Zahler's and Sussmann's criticisms are really directed.

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The six roll mill: experimental observations and computer simulations of the patterns illustrating elliptic umbilic critical point.

SIR,—While I do not wish to perpetuate unwarranted enthusiasm for the ability of catastrophe theory to transform the natural and social sciences, I believe that Zahler and Sussmann (27 October, page 759) have overstepped the bounds of decency in their vehement attack. Let me point out a few specific instances in which they seriously mislead the reader.

A fundamental error that they make is in the statement, "Catastrophe theorists agree that the term 'catastrophe' is reserved for certain kinds of singularity of smooth maps, seven of which have been described and classified elegantly by Thom". I can only conclude from this statement that Zahler and Sussman have not read Thom's work. (There is no reference to Thom in the paper apart from his theorem). Thom describes the catastrophes which Zahler and Sussmann discuss as "elementary catastrophes" but repeatedly makes it clear that there are other kinds of catastrophe as well.

The authors' confusion on this point creates a straw man which they repeatedly flail in their article. For example, juxtapose the quote in the first paragraph of their paper with Thom's general definition of catastrophe and with their restricted one. The quote refers to a general approach to studying questions rather than the repeated use of a specific mathematical theorem, and makes much more sense in the context which was intended.