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Rheology

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deformation
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ultimate tensile strength
viscoelasticity
work hardening

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Definition

Rheology deals with the systematic description of the ways in which materials flow and deform under the action of applied forces.

Basic principles

The forces acting on a body are called the *stresses.* These can be longitudinal (ie tensions or compressions, tending to alter the lengths of lines in the body) or *shearing stresses* (which tend to make neighbouring planes in the body slide over one another), **1**. The changes in dimension and shape caused by these stresses are called *strains.* The complicated tensor mathematics used in rheology is necessary to describe the interplay between longitudinal and shearing effects. The units of stress are those of pressure (ie force per unit area), whereas strain is a dimensionless quantity, being related to the fractional change in length for longitudinal deformations, and the angle of deformation for shear strains.

The type of strain depends fundamentally on the time behaviour of the stress. Thus stresses whose duration is greatly in excess of the so-called *relaxation time* produce liquid-like flow, where the strain constantly increases; the relevant quantity is then the *rate of strain.* However, stresses applied over an interval short compared with the relaxation time produce the constant strain characteristic of a solid. The distinction between solids and liquids is thus not clear-cut, but depends on the duration, relative to the relaxation time, of the stresses acting on the materials as normally encountered. Typical approximate relaxation times are: concrete, 10^6 sec; waxes, asphalt, putty, 10^4 to 10^{-4} sec; air, 10^{-10} sec; water, 10^{-13} sec.

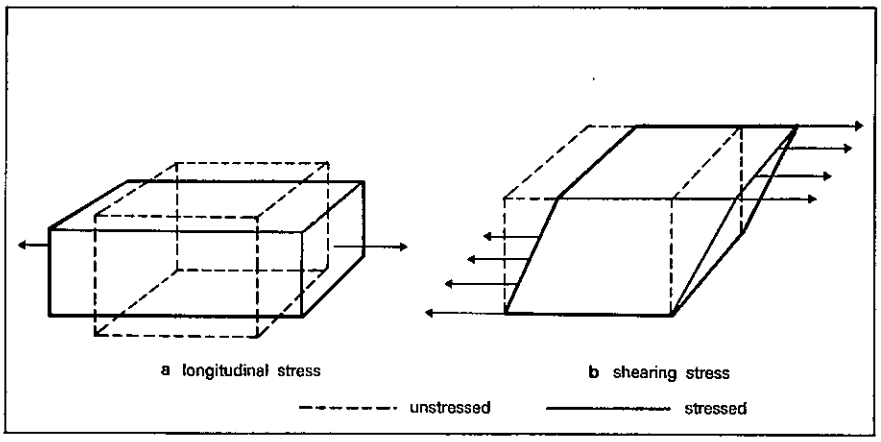
To avoid the complexity of stress-strain relations (or rheological equations) valid for all types of stress and all times, real materials are treated as partaking in differing degrees of the characteristics of three idealised rheological bodies.

Ideal rheological bodies

The elastic solid is characterized by the strain at any instant depending only on the stress at that instant. Examples include rubber, **3a**, cold glass, **3b**, and metals below their yield points, **3c**, provided the stress on these materials is not maintained over times longer than the relaxation time. Special cases are the *Hooke solid,* where the strain is proportional to the stress, and the rigid body of classical mechanics, where the strain is always zero. Schematically, an elastic body can be represented by a spring, **2a**.

The viscous liquid is characterized by the strain rate at any instant depending only on the stress at that instant. Examples include water, oil, hot glass, and ice (when the stresses act for a long time). Special cases are the *Newtonian liquid,* where the strain rate is proportional to the stress, and the perfectly mobile fluid of hydrodynamics, where the shear stress is always zero (the only strict example of this appears to be liquid helium II). Schematically, a viscous fluid can be represented by a dashpot, ie a piston moving through a resisting medium, **2b**.

The plastic solid is characterized by the fact that no strain at all occurs until the stress reaches a yield value. When the body is strained beyond the yield point, the stress varies, but if the stress is removed the material remains deformed and does not revert to its original shape as in the case of elastic strain (this behaviour is called *permanent set*). Examples include polythene, **3d**, and ductile metals, **3c** (the technological usefulness of metals is largely a result of their *ductility,* which contrasts with the *brittleness* of, say, glass, which remains elastic until it breaks). A special case is perfect plasticity, where the strain becomes infinite as soon as the stress is raised above the yield value. Examples are sands, solids and mineral suspensions (the Aberfan tip disaster was caused by plastic slipping).



1 Longitudinal stresses tend to alter the lengths of lines in a body. Shearing stresses tend to make neighbouring planes in the body slide over one another

The distinction between plastic and viscous flow is well illustrated by the different industrial processes necessary to make fine fibres of glass and steel. Hot glass is viscous, and thin filaments can be made simply by pulling the two ends apart. Any attempt to make wires of steel by this method results in the formation of a neck which rapidly narrows down, leading to breakage. Being a plastic material, however, steel can be made into wires by extrusion through a die.

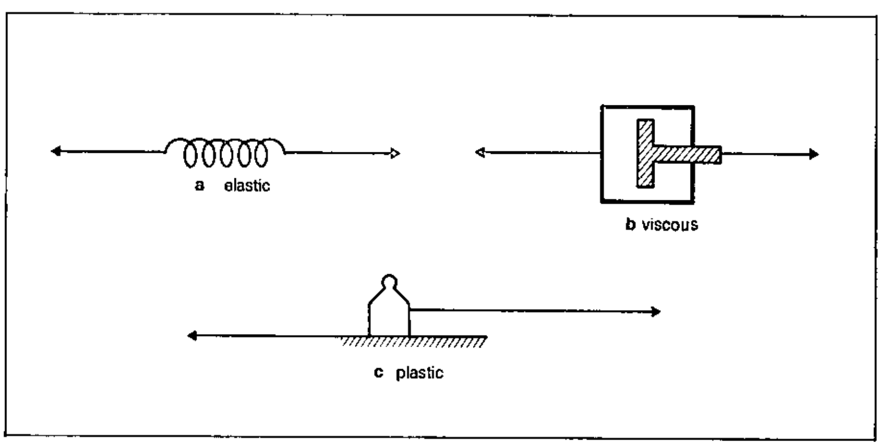
Schematically, a plastic material can be represented by a weight being pulled over a rough surface, **2c**. Because sliding friction is usually somewhat smaller than static friction, this model can describe the drop in stress which often occurs when plastic flow starts.

Real materials

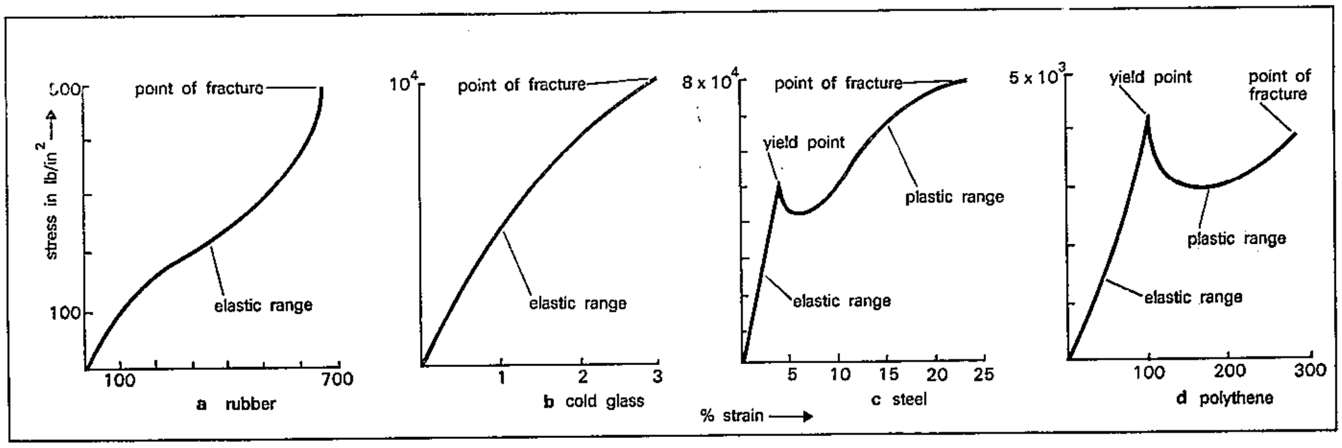
By combining the three basic bodies in different ways, many more complicated types of rheological behaviour can be simulated.

Clays, pastes, oil paints and chocolate all stay rigid until the shear stresses exceed the yield value, and thereafter they flow like viscous liquids, with the strain rate rising with the stress. They are *Bingham materials*, and can be represented schematically by viscous and plastic elements in parallel, **4**.

A different type of complex rheological behaviour is exhibited by glass, steel, concrete and many polymers. If a small stress is suddenly applied and maintained for a long time, the strain instantaneously attains its elastic value, and then slowly rises. The material is said to *creep*. If the stress is then suddenly removed, the initial elastic strain disappears, and the strain achieved during creep gradually relaxes to zero, **5a**. This is *creep recovery*. If, instead, the strain is kept constant, then the stress gradually relaxes to zero from its initial elastic value (this is the reason why tightly-fastened bolts loosen over a period of months, particularly in the hot parts of machines). These are *viscoelastic materials* (see Outline 18), and can be represented schematically by various combinations of dashpots and springs, **5b**. If these elements are connected in series, the result is a *Maxwell liquid*, which shows creep but not creep recovery. If they are



2 Schematic representation of basic types of rheological behaviour. **a** Spring (elastic). **b** Dashpot (viscous). **c** Weight on rough surface (plastic)



3 Behaviour of tensile stress in four common materials as strain is increased continuously

joined in parallel, *a Kelvin solid* results, which does show creep recovery but where the creep reaches a finite value and the sudden changes associated with the elastic strain are absent.

Much work has been done in analyzing still more complex materials (eg flour dough) in terms of the three basic rheological elements.

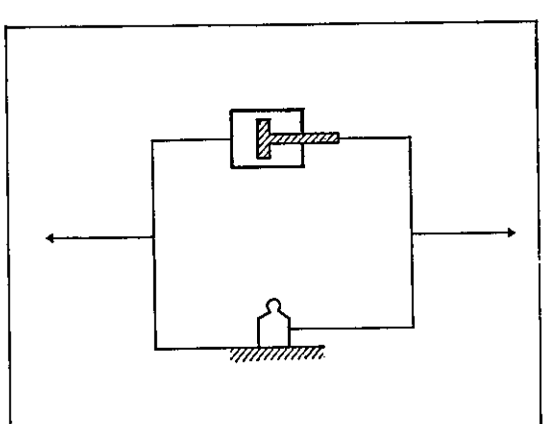
Strength of solids

This is not a quantity to which a single numerical value can be given, and there are in fact three essentially different measures of strength, involving three different features of the rheological curves.

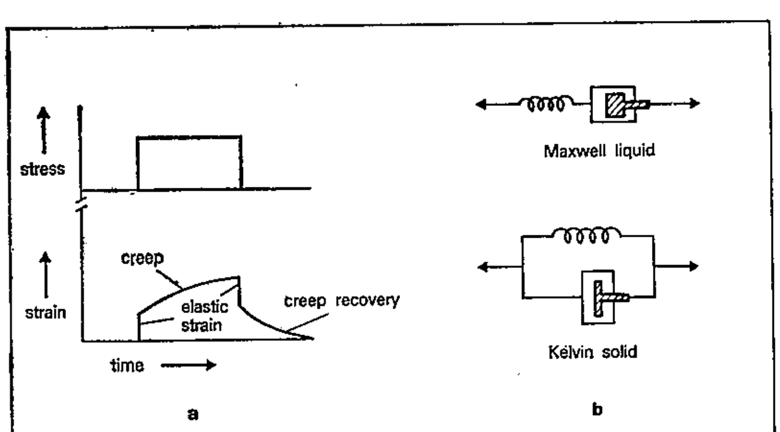
The elastic stiffness is measured by Young's modulus, which is the slope of the stress-strain curve near its origin. Glass and steel have comparable high stiffnesses, and rubber and cloth fibres, like cotton, flax and silk, are flexible.

The ultimate stress at which a material fractures (indicated by the end-points of the curves in **3**) is usually far greater in compression than in tension (this is why masonry structures are designed to be always in compression, and explains the superiority of the arch to the lintel). The basic reason for this asymmetry of strength is that the stresses near the concave parts of the small cracks or steps which always exist in the surface of a material are far larger than the average (applied) stress throughout the material, so that in tension these steps and cracks can open and propagate through the material. In terms of ultimate tensile stress, cloth fibres and steel are strong (about 8×10^4 lb/in^2), and glass is weak (about 10^4 lb/in^2). However, if the surface of glass is carefully polished and kept free from any contact with other solids that would introduce cracks then it can be as strong as steel. Very thin needle-shaped crystals—*whiskers*—can be made to have enormous strength (for instance, 5×10^5 lb/in^2), because the growth steps in such young crystals are small and the stress concentration effect is minimal.

The plastic yield stress in ductile materials (**3c, d**) is in many respects a more useful criterion than that provided by the conventional elastic analysis when it comes to designing metal structures. Plastic flow around



4 Model for a Bingham material such as clays, pastes, oil paint and chocolate



5 Viscoelastic materials. **a** Behaviour under constant stress of finite duration. **b** Maxwell and Kelvin models

the tips of cracks relieves the stress and prevents them from spreading into the body of the material. However, as the strain increases, *work-hardening occurs*, this mechanism no longer operates, and the material eventually breaks, **3**.

Applications and outlook

As the science of materials develops, more kinds of substances are used in industry in an increasing variety of applications. It is essential to rational design that the properties of materials more complicated than elastic solids and viscous liquids be specified in a way that has a sound theoretical foundation.

When the distribution of stresses applied to a material is not uniform (as in buildings and machine parts), then it is difficult to carry out even the traditional analysis which assumes elastic behaviour to calculate the resulting strains. For more complex materials (eg glacier ice), the mathematical difficulties are immense, but progress is gradually being made. An exception is the case of plastic structures, where the procedure is sometimes simpler (see Outline 15).

Major advances are being made on two main fronts in predicting the rheological behaviour of materials from a knowledge of their microscopic structure. The first concerns complex substances, in which a dispersed phase is distributed throughout a continuous medium, where either may be solid or liquid. Examples of such materials are loam, bitumen and concrete. The second major advance concerns crystalline solids, whose ductility or brittleness depends on how easily dislocations can move within them.

Some centres of research or further information

Birmingham University
Bristol University
National Physical Laboratory, Teddington
Newcastle upon Tyne University
Reading University
Royal Aircraft Establishment, Farnborough
Rubber and Plastics Research Association, Shawbury

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