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Measuring the Change in Thickness of the Antarctic Ice Sheet

Is the Antarctic ice sheet growing or diminishing in thickness? This is an interesting question because the change in thickness reflects, in an indirect way, the change of world climate, and also because it directly affects worldwide sea level. It is difficult

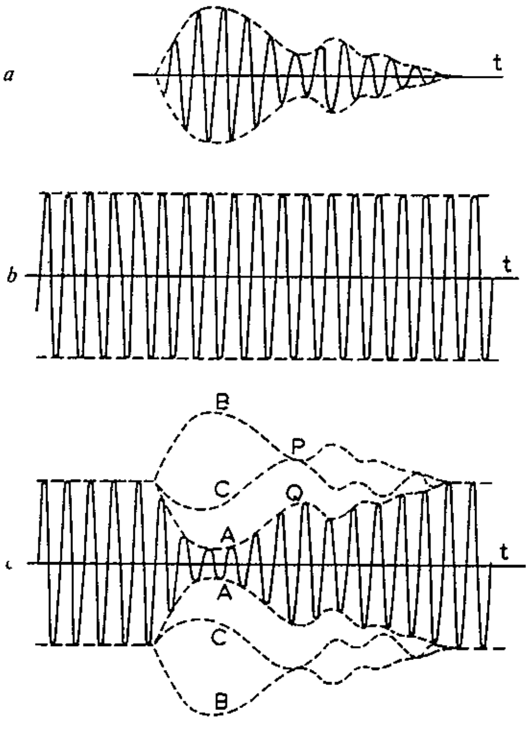

Fig. 1 The echo returned from the rough bed rock of an ice sheet (*a*) is mixed with the carrier wave (*b*) to produce, by interference, a pulse-modulated carrier wave (*c*) that is sensitive to the phase of the echo.

to obtain the answer by measuring the mass balance of the ice sheet, primarly because of the difficulty of estimating the rate of loss of ice by outflow at the margins. A more direct way is to measure the rate of change of gravity at an inland station and try to interpret this in terms of the thinning or thickening of the ice sheet. For example, a steady increase of g at South Pole station between 1957 and 1967 has been well established, but the most recent analysis[1] concludes that 90% of the change can be attributed to the sinking of the station, partly in balance with the normal snow accumulation rate and partly in response to the superimposed load of the snow drift caused by the presence of the station. No disequilibrium of the ice sheet is implied by the gravity observations at South Pole station.

Here we suggest a method of measuring the thickness change, by radio echo methods, that seems to be more accurate than the gravity method and that is free from some of its difficulties. (Other things being equal, a method that measures the relative change of elevation above the bed rock, rather than the relative change in the distance to the centre of the Earth, is to be preferred.) When a radio echo sounder[2] is placed on the surface of an ice sheet the shape of the echo returning from the bed is determined partly by the shape of the outgoing pulse and partly by the "roughness" of the bed[3]. As the echo sounder is moved horizontally the strength of the returning echo changes considerably in distances of the order of the wavelength of the radiation used[4] (typically 5 m in ice) and its detailed structure also changes. This spatial fading pattern has been used[5,6] to provide a frame of reference, finely structured and fixed relative to the bed, against which the horizontal movement of the top surface of the ice may be measured to an accuracy that is at present $\lambda/100=5$ cm, where λ is the wavelength in ice. The fading pattern thus described provides a precise horizontal reference but is very insensitive to vertical displacement; to make it sensitive in the vertical direction we propose to use the phase of the echo in the following way.

The returning echo consists, at least approximately, of a carrier wave (of frequency 35 MHz, say) within an envelope (Fig. 1*a*). Let the transmitted pulse be produced by suitable gating of a continuously-running 35 MHz oscillator, and now add a continuous wave (Fig. 1*b*) from this same oscillator to the returning pulse at the aerial, making the amplitude of the continuous wave slightly greater than the maximum amplitude of the echo (Fig. 2). The effect will depend critically upon the phase relation between the two carrier waves. If they are in antiphase the envelope of the observed signal will be A in Fig. 1*c*, and if they are in phase it will be B. In fact the phase of the echo relative to that of the continuous wave will vary along the length of the echo, slowly near the head of the echo and faster in the tail; the observed envelope will therefore be some curve C, which wanders between A and B, as shown. If the distance H between the aerial and the mean bed now changes by $\lambda/4$, a point on the envelope such as P, where the waves were in phase, will change to Q, with the waves in antiphase. Thus the envelope is now sensitive to displacement in the vertical direction; it is also, of course, still sensitive to horizontal displacement.

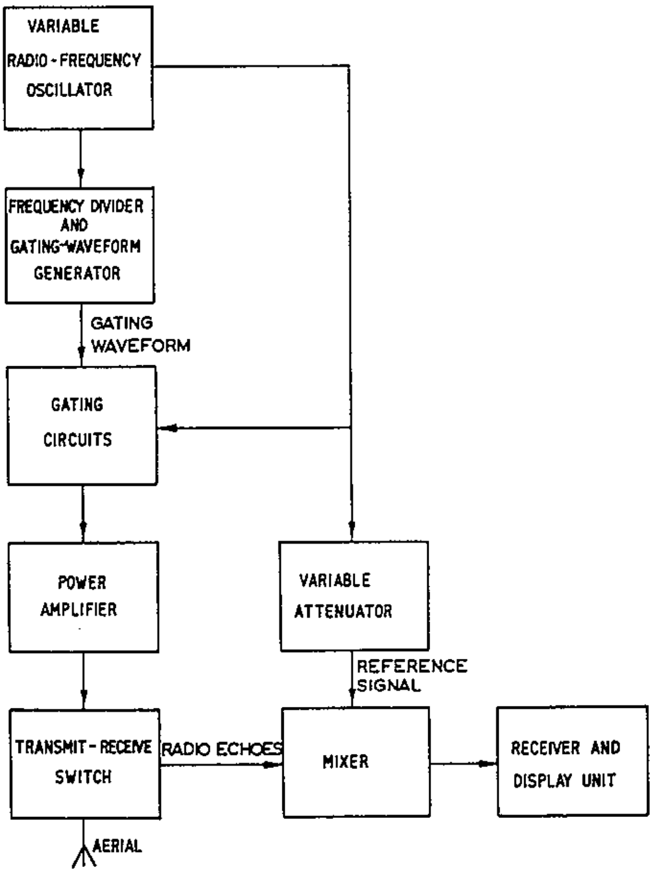

Fig. 2 Schematic arrangement for mixing the returning echo with a continuous wave derived from the same oscillator.

The suggested technique is therefore first to use the envelope of the pulse, unmixed with the continuous wave, to establish position in the plane parallel to the bed, and then to use the envelope, after mixing with the continuous wave, to establish position in the vertical direction (or, instead of using the envelope, one could rectify and smooth). The sensitivity in the horizontal and vertical directions is likely to be rather similar, and so in this way one may effectively set up an (x, y, z) coordinate system, fixed relative to the rock bed and independent of visible landmarks, with a precision of a few centimetres. The displacement of features such as the surface of the snow, a particular firn layer, or markers set in the snow, can then be measured relative to this fixed frame. Of course, the interpretation of the displacement so measured is a separate question: a rise, relative to the rock bed, in the snow surface at a given place could be caused by a recent snowfall, by the horizontal movement through the point in question of a permanent or temporary surface bulge, by snow drift around the station, or by a long-term increase in thickness of the whole ice sheet or of the local part of it. These are matters of interpretation; the present method simply makes the displacement apparent, however it may be caused. By comparison, if gravity changes were used, a vertical sensitivity of 1 cm would require measurements of g to a precision of 3 μgal or 3 parts in 10^9, which is beyond the capacity of present instruments that could be used in this situation; moreover, as Bentley shows[1], the measurements would need a number of uncertain corrections before they could be interpreted as showing vertical movement relative to the rock bed.

In outline, an experiment might go as follows. The aerial is placed at P_1 on the surface of the ice sheet at time T_1. The horizontal fading pattern is observed and then, adding the continuous wave, the envelope C (Fig. 1*c*) is observed. At time T_2 (say 1 yr or more later) the aerial is again placed on the (new) surface of the ice sheet and, by observation of the horizontal fading pattern, a point P_2 is located that lies on the same vertical line as P_1 (P_1 is fixed in space but its position has, of course, been lost). The continuous wave is added, and, in principle, one could relocate P_1 by moving the aerial vertically until the envelope C was regained. In practice it would be preferable to keep the aerial at P_2 and to change the carrier frequency ν slightly, by $\Delta\nu$, say, until envelope C was restored. The number of waves in the path through the ice is $2H\nu/V$, where V is a mean wave velocity in the ice; so, when the same envelope is regained, with the same number of waves in the path, $\Delta(H\nu)=0$. Hence $\Delta H/H=-\Delta\nu/\nu$, where ΔH is the distance of P_2 above P_1. If $\Delta H=1$ cm and $H=3{,}000$ m, $\Delta\nu/\nu=-3\times10^{-6}$, so no remarkable frequency calibration is demanded. (The wave path is $2H$ only for the front of the returning echo; it is longer for the tail, and so the $\Delta\nu$ required will be rather smaller for the tail than for the head of the pulse. This effect is not likely to exceed a few per cent, but it means that the envelope C will not be perfectly regained for any single frequency setting unless P_2 coincides with P_1.) Because the pattern repeats itself when H changes by $\lambda/2$ (say, 2.5 m), the observation time $T_2 - T_1$ should not be so long that there is any ambiguity in identifying the setting where there are the same number of waves in the path. Alternatively, any ambiguity could be resolved by repeating the observations using another basic frequency, or several. The fastest spatial variation in the envelope of the echo occurs in its tail, and this suggests that the tail should be used for horizontal positioning. Any unknown error ε in horizontal position will introduce a corresponding change into the phase of the echo and hence produce an error δH in the value of ΔH that is inferred: thus $\varepsilon/\lambda_f = 2\ \delta H/\lambda$, where λ_f is the horizontal "fading" wavelength for phase. The slowest horizontal variation in the phase of the echo (and in its envelope) occurs in the main body of the echo, where λ_f is of the order of λ or larger[7]; thus, to minimize the error δH we should keep ε small by working on the tail of the echo, so far as signal strength allows, and then, after mixing with the carrier wave, we should keep λ_f large by working on the main body of the echo.

The aerial is placed on the snow surface each time so that changes in its immediate surroundings, and therefore in its radiation properties, are minimized—but our analysis has neglected any effect due to a change in V over the period of observation. It is, of course, important to measure any phase shifts that might arise from a change in the performance of the electronic equipment between the two observation times. But, however much one may minimize such sources of error, the precision of the measurement will ultimately be limited by the observed signal-to-noise ratio (S/N). Phase angles can be measured with a precision determined by this ratio and the corresponding accuracy δH in height change is $\delta H = (\lambda/4\pi)(N/S)$. Thus an accuracy of 1 cm in height change, with $\lambda = 5$ m, would require a signal-to-noise ratio of 40 in amplitude (32 dB in power). The signal-to-noise ratio can be improved by observing many repetitions of a given echo.

An advantage of using pulses rather than continuous waves is that the delay time distinguishes the particular reflecting surface that is being used. This would normally be the rock bed, but it might also be possible to use a reflecting layer within the ice sheet. In this way the (x, y, z) components of the displacement of an internal layer might be accurately determined, and thus one could build up a three-dimensional map of the ice displacements not only at the surface but also within the ice sheet. The method might also be found useful on ice masses smaller than the Antarctic ice sheet—the Greenland ice sheet is an obvious example, but when studying the mass balance of any wide polar glacier it could be helpful to have a fixed reproducible reference frame of the sort this method can provide.

The procedure outlined here has points of similarity with well-established techniques used in other fields. It resembles the Michelson interferometer used at optical frequencies, but with the two important differences that the reflecting surface is rough and that the use of pulses rather than continuous waves gives range information. It is more like making a hologram: as in holography, the system takes a wave from a rough surface, adds a reference signal and explores the spatial variation of the resultant signal (in the horizontal plane, say), but the use of pulses is a point of difference. From another point of view one might see an analogy with the use of radar to infer the velocity of a moving target by measuring the Doppler frequency shift between the transmitted and the received wave. In the Antarctic situation, if the velocity were constant, the phase change (a fraction of a cycle) with time (perhaps 1 yr) could be regarded as a Doppler frequency (a shift of 1 part in 10^{17}). But the analogy is formal only: the method essentially measures a phase change rather than a frequency and therefore displacement rather than velocity. The general principle of mixing the echo with the carrier wave could be used to improve the range accuracy of any echo sounder; an ultrasonic flaw detection apparatus is one example.

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