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Fine structure in caustic junctions

EVERYONE has seen the changing patterns of bright caustic lines traced on the sea bed or on the bottom of a swimming pool[1,2]. The caustics are formed when sunlight is focused after refraction by smooth waves on the water surface; similar caustics can result from reflection. Here we draw attention to one feature of these patterns: the frequent occurrence of junctions at which three caustics appear to meet (Fig. 1).

It might seem that this is not worthy of comment—after all, the stability of triple junctions under perturbation makes them widespread in many natural systems of lines and surfaces[2,3] such as mud cracks, foams, and the markings on giraffes. But triple junctions in caustic patterns would be surprising. The reason is that, except in special circumstances, the forms of caustics are restricted to be 'elementary catastrophes'[4–6] and in two dimensions (i.e. two 'control variables') these are smooth curves ('fold' catastrophes) which may have cusp points: triple junctions occur nowhere in the list of elementary catastrophes. The qualification 'except in special circumstances' implies the assumption of genericity that underlies catastrophe theory. It means in this case the absence of high symmetry in the wavefront whose normals envelop the caustic and is certainly satisfied for the random water waves considered here.

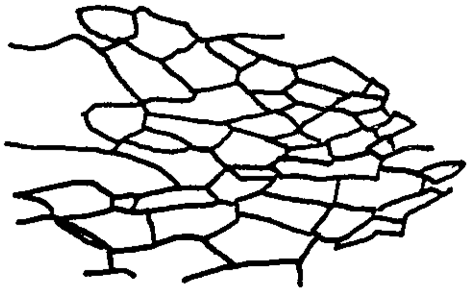

Fig. 1 Caustics formed by a wavy water surface (after Minnaert[1]).

These considerations suggest that the triple junctions are illusory. In fact they are the appearance under low resolution of a caustic structure with fine detail on several levels. The following argument shows that this structure involves the meeting of six lines rather than three. Below each 'ridge' on the water surface is a caustic surface with two sheets that meet along a cusped edge (Fig. 2); if the sea bed intersects this surface it will generally do so in two almost-parallel lines (these are easily seen in sunlight in, say, 2 m depth of water on which the waves are long-crested). The ridges meet in threes on the water surface, and the associated meetings of three line pairs on the sea bed are the 'triple junctions' under discussion.

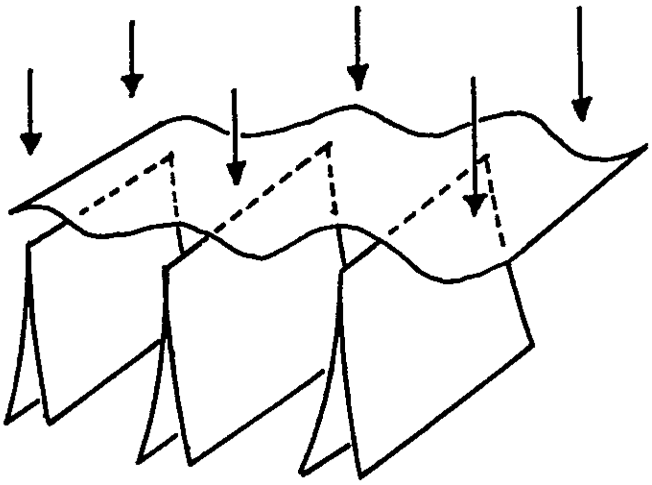

Fig. 2 Line pairs where caustic surfaces from ridges intersect the sea bed.

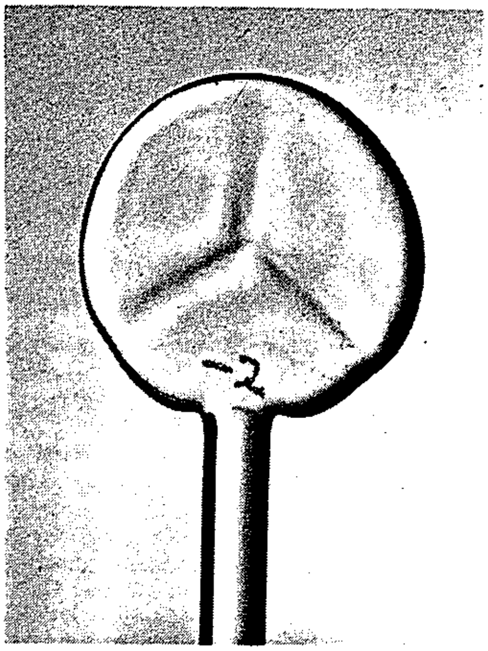

Fig. 3 Glass 'lollipop' on whose surface three grooves meet.

To study the details of such a junction *'in vivo'* would be difficult in view of its rapid motion and the blurring caused by the 0.5° divergence of rays from different parts of the Sun. Therefore conditions near the junction of three ridges were simulated *'in vitro'* on the surface of the disk of a glass 'lollipop' (Fig. 3) formed in the molten state. The diameter of the disk was 40 mm. White light from an illuminated pinhole was viewed through a microscope after being

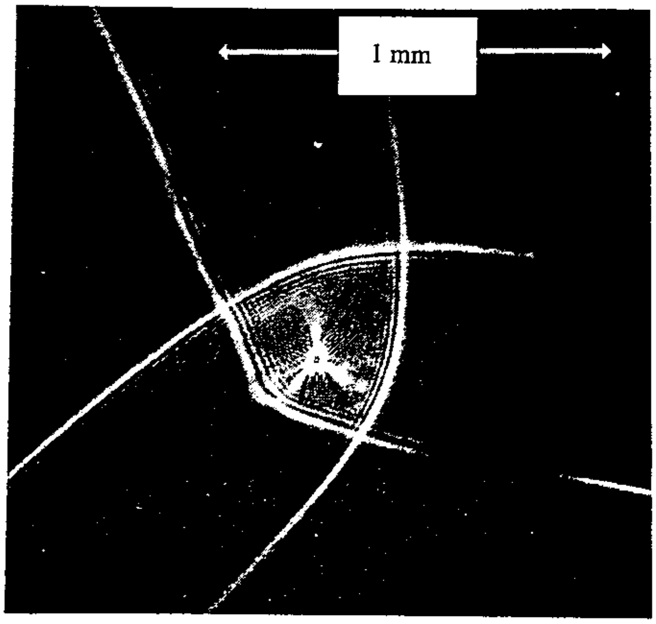

Fig. 4 Typical fine structure in caustic junction, formed by refraction of white light from a small distant source through the 'lollipop' of Fig. 3 and photographed with the aid of a microscope.

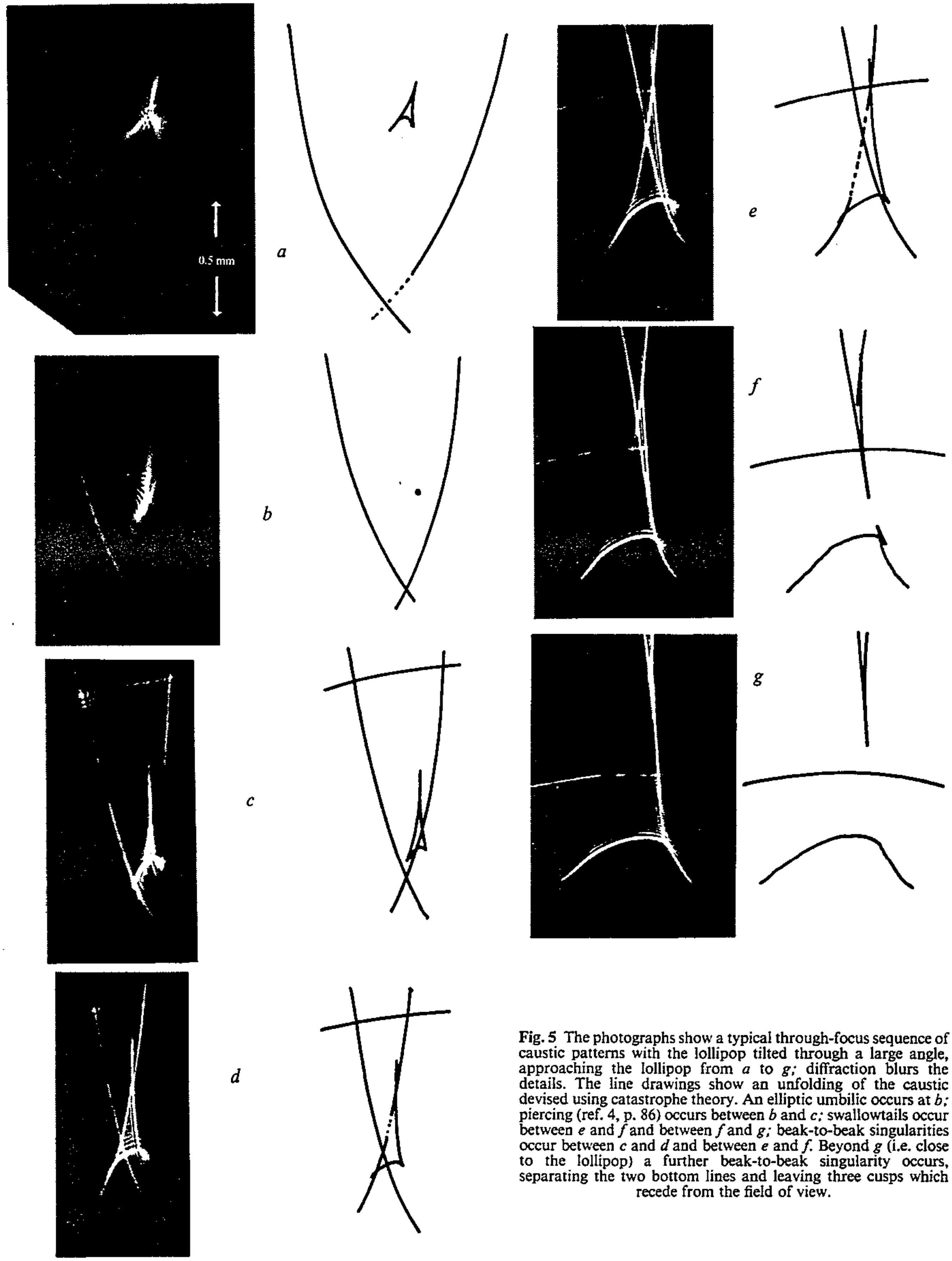

Fig. 5 The photographs show a typical through-focus sequence of caustic patterns with the lollipop tilted through a large angle, approaching the lollipop from *a* to *g*; diffraction blurs the details. The line drawings show an unfolding of the caustic devised using catastrophe theory. An elliptic umbilic occurs at *b*; piercing (ref. 4, p. 86) occurs between *b* and *c*; swallowtails occur between *e* and *f* and between *f* and *g*; beak-to-beak singularities occur between *c* and *d* and between *e* and *f*. Beyond *g* (i.e. close to the lollipop) a further beak-to-beak singularity occurs, separating the two bottom lines and leaving three cusps which recede from the field of view.

refracted by the disk. (It was easier to produce grooves on the glass surface rather than ridges, so that the caustics were virtual rather than real.) In this way the three line pairs were easily resolved and typical patterns do indeed show the smooth curves and cusps anticipated on the basis of catastrophe theory (see Fig. 4). Note the inner cusped 'triangle', for this provides the key to the understanding of the higher-dimensional caustic structures now to be discussed.

Altering the focus of the microscope introduces an addi-

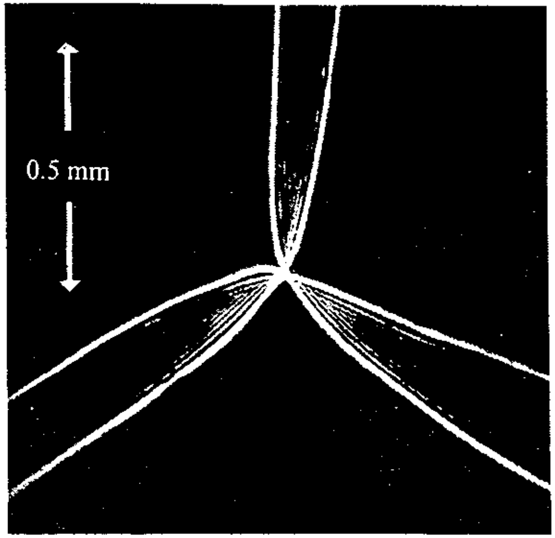

Fig. 6 Singular section of a higher catastrophe.

tional control variable which enables the three-dimensional caustic surface to be explored in two-dimensional sections. In nature this new variable could represent time. A 'through-focus' sequence of patterns is shown on Fig. 5*a–g*. The complicated changes in caustic structure must be governed by catastrophe theory with three control variables. In our experiments the finest details of some of these changes were obscured by characteristic diffraction effects; the intensity and linear scale of the diffraction effects increase, for fixed wavelength, with the order of the singularity. According to catastrophe theory, cusp points may appear and disappear in pairs, either at beak-to-beak or lip singularities[4,5] (where the plane of focus touches a cusped edge of the caustic surface) or at swallowtail catastrophes (singular points of the caustic surface). Guided by these principles it was possible to devise a series of catastrophes (the line drawings on Fig. 5) to account for the transformations on the photographs. The line drawings show the patterns that would be formed in the limit of geometrical optics where the wavelength approaches zero. There are further singular points of caustic surfaces, associated with elliptic and hyperbolic umbilic catastrophes, but since these catastrophes are symmetric in their unfoldings they do not result in changes in the topology of the two-dimensional patterns; an elliptic umbilic occurs on Fig. 5*b* as the cusped 'triangle' shrinks to a point and expands again. All this behaviour is typical (generic): no special care is required in orienting the lollipops or in manufacturing them (other than making them smooth). Qualitatively identical phenomena were also observed in reflection.

The tilting of a lollipop provides two more control variables, θ_1 and θ_2, say. By changing θ_1 it is possible to find a through-focus sequence where the cusped triangle shrinks to a point just as it reaches one of the caustic lines; this is a parabolic umbilic catastrophe point P on the three-dimensional caustic hypersurface in the four-dimensional control space C. The sequence on Fig. 5 constitutes a three-dimensional section S of C, and the fact that P lies close to S explains the small scale of detail in the patterns. In Thom's terminology[4] the parabolic umbilic is the 'organising centre' for caustics near P and its unfolding justifies the sequence of patterns in the line drawings of Fig. 5. It is also possible to make two swallowtails coincide; this is a butterfly catastrophe. Finally, by also changing θ_2 this butterfly and parabolic umbilic can be made to coincide at one stage in the through-focus sequence; the two-dimensional pattern (Fig. 6) is then a singular section of the higher-order catastrophe, presumably involving five control variables, which organises all the triple junction caustic patterns discussed here.

We draw two conclusions from this study. First, nature's line patterns are not all of the same sort; the triple junctions generic in mud cracks cannot occur with caustics. Second, the geometrical optics of cylindrically symmetric artefacts such as telescopes, where departures from the ideal point focus are treated as 'aberrations', is very different from the geometrical optics of nature, where the generic forms of caustic surfaces are governed by the mathematics of catastrophe theory.

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