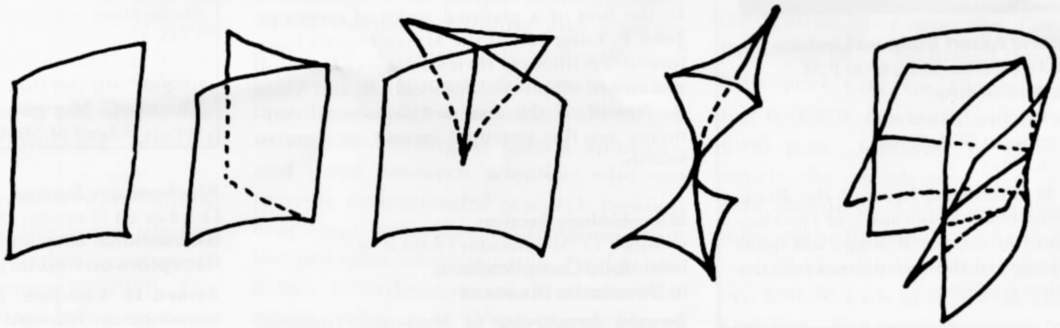
Forms of Light

by Michael V. Berry

The sea's sparkles and stars' twinkles are natural examples of complex geometry

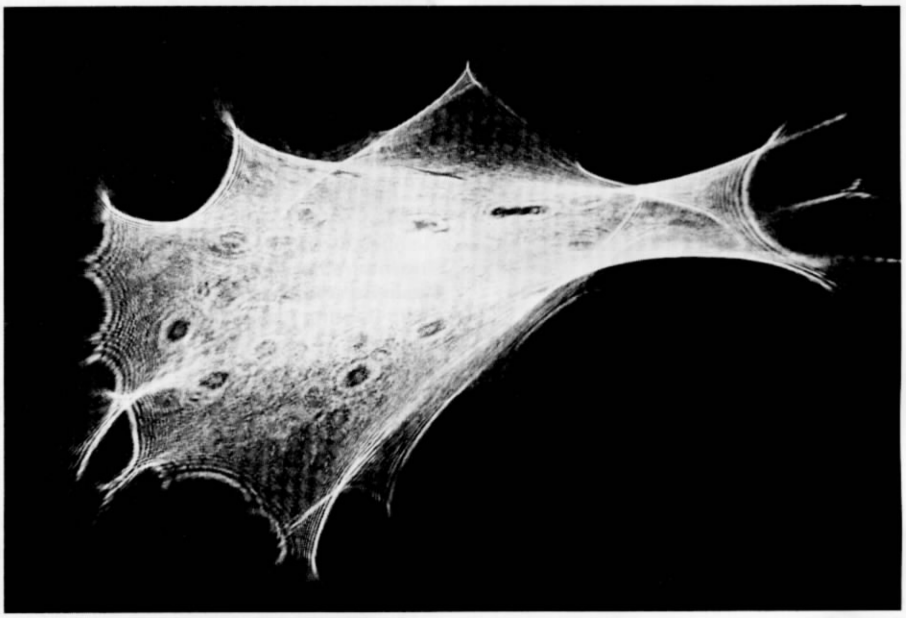


Schematic representations of the five basic caustic catastrophes in three-dimensional space.

Sparkling reflections on a sunlit sea; bright lines dancing on the sand below; a bright star twinkling low in the sky on a frosty night; the rainbow's graceful arch—all these are nature's actualizations of geometric forms classified by the French mathematician René Thom as part of his "catastrophe theory." Some uses of this new mathematics are controversial, but the applications to optics are securely founded on the laws governing light rays.

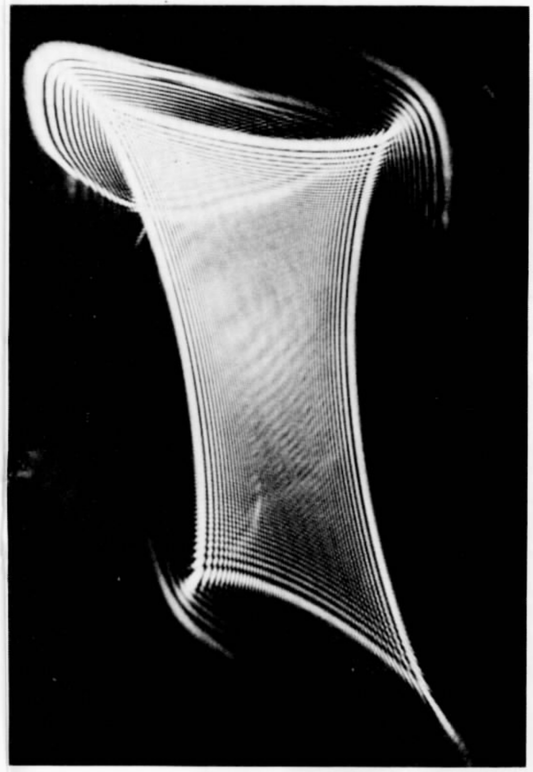
Thom's theory classifies optical patterns produced by focusing. In conventional optics the most familiar focus is the isolated point at which rays intersect after passing through a perfect lens. Such a focus is unstable. It is destroyed by the slightest disturbance—deformation of the lens, movement of the object or refraction by air currents. Instead of passing through a single point, the rays then envelop complicated surfaces in space. These focal surfaces are called caustics (from the Greek word for burning). In catastrophe optics, attention is concentrated on the stable caustics, which, under disturbance, distort but maintain their essential form. Almost all arrangements of smooth refracting and reflecting objects will produce stable caustics. Therefore they can be expected to occur in nature, for instance in refraction by wavy water or turbulent air,

Michael V. Berry is Reader in Physics at the University of Bristol, England, and his main research is on topological aspects of wave motion. His most recent book is Principles of Cosmology and Gravitation *(Cambridge University Press, 1976). In 1978 he was awarded the Maxwell Medal and Prize for theoretical physics.*



The "cusp" catastrophe (above), caused by refracting light through an irregular water-drop "lens," is familiar to people wearing glasses in the rain.

The complicated focal patterns of "compounded" caustics are revealed in this image (right) of a "hyperbolic umbilic" catastrophe, projected by a laser beam refracted by irregular bathroom window glass.



a far cry from the controlled symmetry of the optical technologist's world.

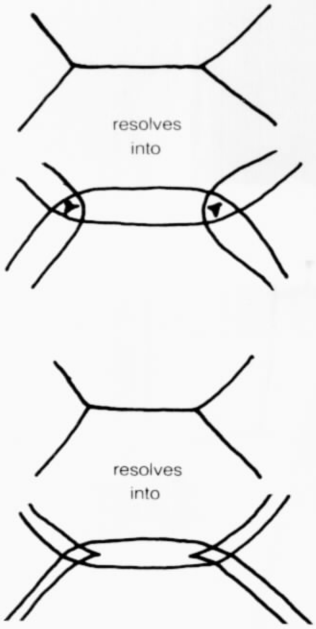
Elemental Forms
The remarkable result of Thom's theory, unsuspected in the preceding three centuries of geometrical optics, is that only five sorts of caustic or "catastrophe" are stable in three-dimensional space. These catastrophes are "elemental forms." Through them, a great variety of focal patterns can be understood.

The sparkling of the sea on a sunny day results from a succession of the simplest catastrophes: fold surfaces. To see this, one must realize that the water produces images of the sun whenever the waves have the right slope to reflect light into the eye. As the waves change, the images move. At certain instants ("twinkles") two images meet and annihilate, or two new ones are born. This happens whenever the water has the right curvature to focus light into the eye. Above the sea there is a rapidly moving pattern of caustic surfaces, and a twinkle corresponds to one of these passing the eye. It is the rapid succession of twinkles that causes the sparkling appearance of the water.

David Hockney's 1971 painting, Portrait of an Artist (Pool with Two Figures) *features a caustic network produced by refraction. Photo courtesy of Knoedler Gallery, London.*

Another example of fold caustics in nature is the rainbow. Each raindrop refracts sunlight, producing a fold catastrophe. The rainbow's colors are caused by dispersion. Drops at slightly different angles produce different colors.

Next in the hierarchy is the cusp catastrophe. This is a familiar sight on coffee in a cup in which the inner surface is illuminated, by sunlight, for example. People wearing glasses in the rain often see cusped caustics in the images of distant lights refracted by irregular droplet "lenses." The droplets are irregular because the glass is dirty, and this has led the British chemical company, ICI, to explore the feasibility of using the caustics to monitor the uniformity of wettability of the coated surfaces they manufacture, one example being non-stick frying pans.



Catastrophe Fine Structure: Two resolutions of "impossible" triple junctions indicate that they are actually made up of combinations of simpler forms.

Computer simulation of part of an "elliptic umbilic's" interference pattern (left), compared with an image generated by a laser beam refracted by a triangular-shaped waterdrop "lens" (right).

Hidden Detail

On large scales, the "elemental" catastrophe caustics can combine into "compounds," in the form of complicated focal patterns. Often the full structure of the caustics is obscured by incoherent illumination or rapid motion. In such cases catastrophe theory gives a powerful method for unraveling the underlying detail. For example, the caustic surfaces produced by refraction of sunlight by water in a swimming pool intersect the floor of the pool in a network of bright lines.

The appearance of triple junctions in these patterns seems to violate catastrophe theory. The meeting of three caustic surfaces along a line occurs nowhere on Thom's list of stable forms. However, theory predicts, and experiment confirms, that the junctions can be resolved into a fine structure which shows that the apparent three-line junctions are actually composed of several other forms.

Catastrophe theory contributes much more to optics than a list of the focal geometrics of light rays. On fine scales, where the wavelength of light cannot be neglected, each type of caustic is decorated with a vivid and characteristic interference pattern. This means that Thom's classification of stable caustics can be transformed into a classification of the stable patterns formed by short waves. These are the "diffraction catastrophes."

The fold diffraction catastrophe was analyzed by the English astronomer Sir George Airy in 1838 and applied to the intensity of light in rainbows. The cusp catastrophe was studied by the English applied mathematician Thomas Pearcey in 1946. The higher diffraction catastrophes are being investigated now.

Twinkling Stars

To date, the most far-reaching applications of catastrophe optics is to random waves. Refraction by turbulence in the atmosphere can be strong enough to focus starlight onto caustic surfaces. Individual caustics can be seen crossing the defocused image of a bright star viewed near the horizon through a powerful telescope. The twinkling of the stars is caused by these caustics passing the eye—just as in the case of sunlight sparkling on the sea. The light intensity received at any point fluctuates wildly. It turns out that these fluctuations depend on the wavelength according to complicated scaling laws involving the whole hierarchy of catastrophes—not just those on Thom's list but an enormous extension of the classification derived by the Russian mathematician Vladimir Arnol'd.

Catastrophe theory, combined with experiment and computation, is helping us to understand a whole new realm of optical phenomena. These have the pleasant property, unusual in physics today, of existing not only in the laboratory but also in nature, where they can be seen by everybody. □