Mathematics to describe shapes

The fractal geometry of nature
by Benoit Mandelbrot, *Freeman, pp 460, £22·75*

Michael Berry

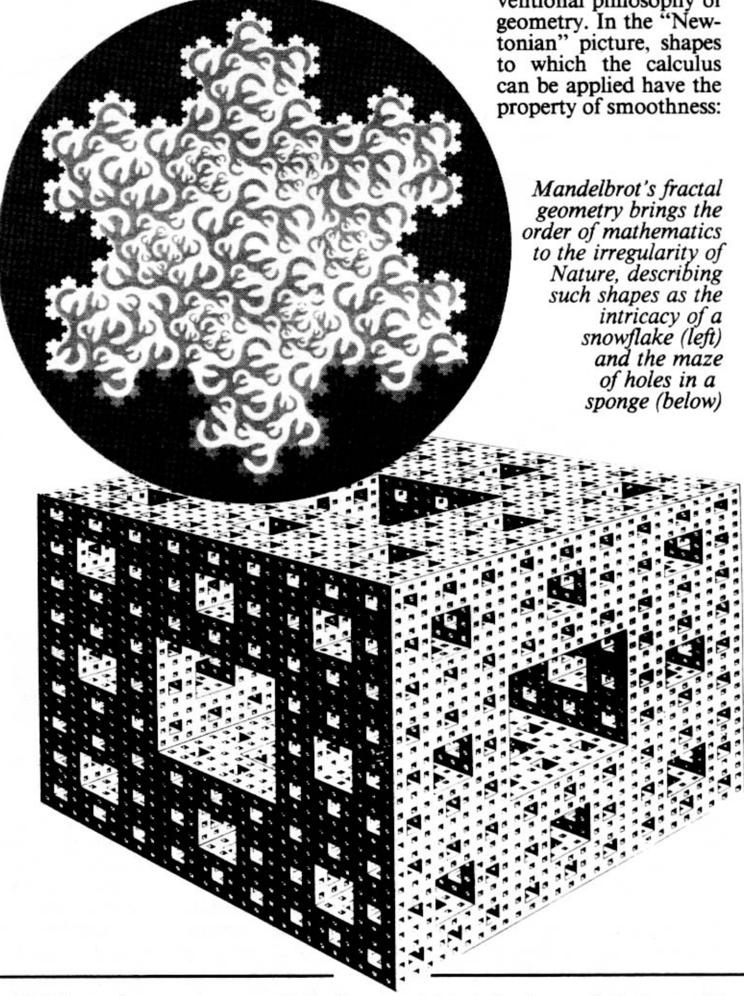
FRACTAL geometry is one of those concepts which at first sight invites disbelief but on second thought becomes so natural that one wonders why it has only recently been developed. Benoit Mandelbrot's central idea, presented here in a revised and expanded version of an essay first published in English five years ago, is that many of Nature's forms have irregularities so extreme that they are best described not by the one-dimensional curves and two-dimensional surfaces of conventional geometry but as intermediate shapes ("fractals") whose dimensionality need not be a whole number. For example, a coastline is a curve whose length (between any two points) increases when measured more accurately so as to include its ever-finer convolutions round bays, headlands, cliffs, boulders, rocks, pebbles, etc, and on any reasonably simple model the length is infinite. On the other hand, the coastline's area is zero, so neither a one-dimensional nor a two-dimensional picture is appropriate, and in fact the way in which the length increases with resolution suggests a dimension D of about 1·2.

The mathematics of sets of points with fractional dimensionality was developed in the early years of this century, but associated geometric objects were considered as "pathological" and not corresponding to anything in Nature. Mandelbrot's massive and single-minded achievement has been to convert this abstract formalism into a flourishing branch of applied mathematics, in three ways.

First, he has enriched our geometric imagination by recognising and systematically exploring a wide range of fractal shapes to augment the familiar (Euclidean) circle, triangle, sphere, cube, etc. With computer graphics of stunning beauty, we are conducted through the weird world of Koch curves, Seirpinski carpets, Menger sponges, Fatou dusts, self-squared dragons, and Apollonian gaskets. All these shapes possess a hierarchical structure extending to infinitely small scales.

Secondly, he demonstrates that fractals are good models for an impressive variety of natural objects. These include landscapes (surfaces with $D \sim 2{\cdot}2$), networks of rivers ($D \sim 2$), lung branches ($D \sim 2{\cdot}9$), and blood vessels ($D \sim 3$), cloud perimeters (curves with $D \sim 1{\cdot}3$), places where energy is dissipating in turbulent fluids ($D \sim 2{\cdot}55$) and, on the largest scales, the hierarchies of clusters and superclusters of galaxies (dusts with $D \sim 1{\cdot}23$).

Thirdly, he emphasises that fractals imply an unconventional philosophy of geometry. In the "Newtonian" picture, shapes to which the calculus can be applied have the property of smoothness: the more they are magnified, the simpler they get (in the limit, curves can be replaced by their tangents). By contrast, magnifying a fractal reveals finer levels of structure *ad infinitum*: the whole structure is contained in microcosm near any point.



Mandelbrot's fractal geometry brings the order of mathematics to the irregularity of Nature, describing such shapes as the intricacy of a snowflake (left) and the maze of holes in a sponge (below)

So far the subject of fractals has been almost entirely descriptive. It is true that to discover that an object has a particular fractal dimension D is a valuable addition to knowledge and replaces earlier imprecise characterisations such as "spotty", "stringy" or "lumpy". But this is only a first step to scientific study, and two questions naturally follow: having found D what use is it? And what is the reason for this D and not some other (or, more generally, what is the origin of hierarchical structure)?

With regard to the first question, time will tell; my own experience is that knowledge of D can open the door to the study of several physical processes involving the fractal structure. As to the second, there are two fractals in physics whose D is fairly well accounted for, namely the "Brownian" drift of a small particle jostled by molecules in a fluid, which is an erratic curve with $D=2$, and the hierarchy of density fluctuations in a fluid at the critical point where it cannot properly be considered as liquid or gas.

It must be realised that fractals are hierarchies with the particular property that the successive levels are geometrically similar (either exactly or on the average). There are many non-fractal hierarchies in which successive levels differ qualitatively. The living hierarchy of ecosystem, species, individual, organ, cell, nucleotide, molecule is an example.

Mandelbrot's essay is written in a personal, intense and immediate style. Technicalities do not intrude but are sufficient to prompt serious research. There is an extensive bibliography and fascinating biographical sketches of the often eccentric scholars who anticipated fractals. This is an important book from which scientists can benefit and which lay people can enjoy. □

Who is responsible?

Scientists, the arms race and disarmament
A Unesco/Pugwash symposium edited by Joseph Rotblat
Taylor and Francis, pp 323, £9·50

Robin Clarke

THIS informative and often horrifying volume results from a Unesco symposium held in Ajaccio, France, in February 1981. Some 20 distinguished scientists give their views on the involvement of scientists with war in general and the arms race in particular. Also included are the conclusions and recommendations of the symposium.

The basic facts, of course, are by now fairly well known: that something like 40 per cent of all research and development worldwide is undertaken for military ends. This makes military research and development by far the most important scientific activity—important in numbers of scientists involved and money spent; it is also the most important in that it threatens the annihilation of more people than any other human activity.

Because of this, it is alleged, the arms race is fuelled into ever more expensive, dangerous and faster spirals. While the motivation for the arms race is political, the scientist himself (and indeed herself) is the mainspring of its momentum, sometimes wittingly, sometimes unwittingly. One of the conclusions of this symposium was that "There is a growing belief that the momentum of the arms race is determined by the actions of scientists. This belief is exaggerated; a multitude of factors, interacting with each other, is involved . . ."

However, the recommendations go on to prove themselves of sterner stuff. They come close to enjoining all scientists to refuse to engage in military research but then back off at the last moment. Instead they implore scientists to "ponder on the social implications of their work" and to become actively involved in the effort to stem the arms race. To help them to do so they list 12 tasks for scientists, such as participating in research on the economic effects of disarmament and urging editors to provide space in their journals for discussion on disarmament issues.

The 18 major contributions are themselves fairly impressive. I particularly liked Francesco Calogero's analysis of the dynamics of the nuclear arms race and John Ziman's discussion of the basic principles involved in talking about social responsibility among scientists. There is an informative chapter on new conventional weaponry, and an analysis of the arms race from the Third World viewpoint.

All in all, this is a commendable book, packed with sound information and wise opinion. One underlying issue, however, troubles me. Throughout there is the implication that to partake wittingly in an arms race is a perversion of science—as the conclusions say, "the role of scientists in the arms race is of crucial importance. This role . . . is contrary to their traditional calling."

Is it? Traditionally, scientists have been involved in war since the year dot. I suspect what the conclusion wants to imply is that there is something in the philosophy of science which says that science should be used for good things, and that war is not one of these. This, of course, is nonsense. On the contrary, the opposite view is intellectually more compelling. As Herbert Marcuse once said, "If the needs of science so perfectly match those of the Pentagon . . .". If nearly half of all science has to do with the military, it is surely tautological to refer to this as a perversion of science. On the contrary, it *is* science.

I doubt, therefore, that scientists engaged in military research are being irresponsible to science. I do not doubt, however, that they are being irresponsible to the world, to humanity and to life on Earth. But that is a very different matter. □