REVIEW

Sisters in symmetry

Symmetry: Unifying Human Understanding
edited by István Hargittai, *Pergamon, pp 1064, £76·50*

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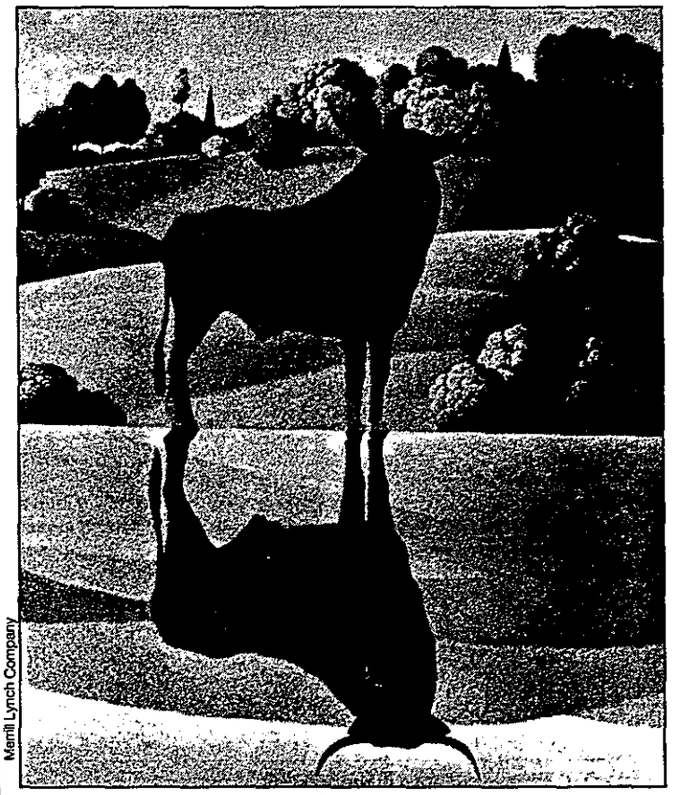
DRIVEN by mutual repulsion, my mother and her six sisters colonised the globe from Whitechapel to Wellington, thereby solving (more or less) the geometrical problem of placing seven points on a sphere so as to maximise the separation between neighbours. Now I learn from the essay by L. Fejes Tóth in this collection that this is the $n=7$ special case of the "inimical dictators problem", whose solution is for the sisters (or dictators) to live at the vertices of a symmetric dipyramid. The more general problem is, in turn, one from a wide class where symmetrical configurations arise from requiring some quantity to be as small or as large as possible.

This massive volume consists of nearly 70 articles, all concerned with some aspect of symmetry. The editor, István Hargittai, interprets the concept in the widest sense: contributions range from the conventional mathematics of spatial arrangement (such as the inimical dictators) through physics, chemistry, biology and geology to dance, the visual arts, music and literature. Some papers represent the disciplined imagination at its best, others appear to be hasty rehashes of the authors' previously published work, and the worst are pretentious nonsense.



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Antisymmetry focuses the attention

In several papers, Mackay takes a broad view of crystallography. He explains how the conventional approach, in which patterns are seen as the result of a group acting on a motif (what operations leave the arrangement invariant?), is being replaced by the more general idea of cellular automata, where large-scale structure arises from the interaction of neighbouring elements. He also gives what, to my surprise, is the only discussion in the book of the recently discovered "quasi-crystals", which have fivefold symmetry and (therefore) no long-range order.

There are several "make-your-own-pattern" papers. Dunham explains with computer programs how to tile the hyperbolic plane; this is not flat like the Euclidean one but curved like a saddle, leading in the representation used here to infinite repetitions of the motif within a circle, getting smaller near the edge. Schattschneider shows how to create black-and-white coloured tilings of the plane.

Hilton and Pedersen give some elegant examples where logical and algebraic symmetry can simplify mathematical reasoning. There is an introduction by Kappraff to the fractal geometry of hierarchical structures; this is clear enough but contains nothing that Mandelbrot, who says it better, has not already published elsewhere. The discussion by Blackmore of the mathematics of chaos is too abstract and too brief to be of use to the newcomer, and is in any case out of place because it does not deal with symmetry in any sense.

A thorough and broad treatment of the symmetry of biomolecules by Vainshtein is particularly valuable because it emphasises the importance of regular structures occurring at different levels of organisation. The mysterious fact that molecules can occur in left- and right-handed forms (chirality) is discussed in a geometrical way by Sokolov, but the way in which handed molecules originated is not considered at all.

Two articles about Moiré patterns complement each other nicely. Giger deals with the mathematics, showing how to construct sets of parallel curved lines which superpose to give the contour lines of any smooth function of two variables. Witschi emphasises the perceptually disorientating effects (not easy to explain) of Moirés, especially moving ones, and shows photographs of many of his beautiful Moiré art objects.

Inevitably, several articles deal with the symmetries of Moorish and other ornaments such as wall tilings and textiles. The outstanding paper here is by Grünbaum, Grünbaum and Shepard. They show that, contrary to the claim often made or implied, the Moorish artists did not exploit all the possible symmetries of plane patterns. For example, they employed only 13 of the possible 17 "wallpaper groups" in which colour is ignored; when colour is taken into account, the number of patterns not exploited is much greater. Washburn points out the strange fact that pre-Columbian textiles are often coloured in ways which are incompatible with their underlying structural symmetry. (I notice a related phenomenon in the overhead projector transparencies that I prepare to illustrate lectures: different-coloured pens are used purely decoratively, in ways quite unrelated to the content.)

A carefully argued paper by Johnson makes the claim that in *Paradise Lost* Milton employs the Fibonacci approximations 5/8 and 8/13 to the golden section, as a mathematical symbol of theodicy (explaining the ways of God to men). He is well aware of the dangers of such interpretations: "In literary scholarship, the discovery of a symmetrical pattern is not in itself particularly interesting; for the sorts of patterns one encounters are not mathematically complicated and, without a clear relationship to the verbal content of a work, are either of no value to the reader or give the impression of having been arbitrarily or even irresponsibly imposed. In these matters, then, the realization of poetical purposes in mathematical symmetry is our aim . . . " These wise remarks are worth keeping in mind while reading some of the other articles about symmetry in the arts, such as the one by Pavlović and Trinajstić, which solemnly tells us that " . . . the symmetry or asymmetry characteristics of letters apparently has little effect on the probability of their occurrence".

Such nonsense apart (and there is much of it, and worse), this is a stimulating collection of the most varied ideas, with many useful sets of references, well produced and abundantly illustrated. □