THE AHARONOV-BOHM EFFECT IS REAL PHYSICS NOT IDEAL PHYSICS*

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The more celebrated of the two effects described by Aharonov and Bohm (1959) is that the behaviour of a quantum charged particle is modified by magnetic flux Φ threading a region from which the particle is excluded. They solved Schrödinger's equation for the scattering of a plane wave of particles by a thin infinite impenetrable cylinder containing a solenoid generating Φ (considered as concentrated onto a single flux line), and showed, as have later analyses for cylinders that are not thin, or for different geometries (e.g. tori) that observable properties do depend on Φ. More precisely, they depend on the quantum flux parameter

$$\alpha \equiv q\Phi/h \tag{1}$$

where q is the charge, the dependence on α being periodic with period unity. A thorough review of the theory of the Aharonov-Bohm (AB) predictions, and experiments carried out to test them, has been written by Olariu and Popescu (1985).

Central to all AB theories is the assumption that the vector potential $\underline{A}$ ($\underline{r}$) in the accessible region, on which the quantum mechanics of the particle depends, has its gauge freedom restricted by the flux in the inaccessible region, according to the Stokes condition

$$\oint \underline{A} \cdot d\underline{r} = \Phi. \tag{2}$$

(Even in the hydrodynamic formulation of quantum mechanics (Casati and Guarneri 1979) where the quantum evolution equations depends on fields rather than potentials and so do not involve Φ, $\underline{A}$ appears in a circuit condition on the Madelung quantum velocity field.) It is precisely the legitimacy of (2) that is disputed by objectors to AB theory (Bocchieri and Loinger 1978). If the flux really is inaccessible, how can an 'outside' observer, wishing to apply quantum mechanics, know its value (apart from doing an AB experiment) and thereby implement the constraint (2)? Without a knowledge of Φ, no quantal calculation can begin: for the scattering of particles by an infinite hole through the universe, quantum mechanics is incomplete.

* This note combines the substance of a statement introducing, and remarks contributing to, a round-table discussion on the Aharonov-Bohm effect.

In assessing the weight of this objection, it is useful to recall the familiar fact that hasty idealization often obstructs the application of theory. The remedy is always the same: to consider the ideal system as the limit of more realistic ones, to which the theory can be consistently applied. Sometimes the conclusion is that there is no unique limit, because behaviour depends on circumstances. An example is the Newtonian forces exerted on the ground by the four legs of a chair; these forces are indeterminate in the limit where material deformations are neglected, and in fact depend on the details of these deformations. In other cases, unique behaviour does emerge as the idealization is made. An example is the reflection boundary condition on the quantal wavefunction $\psi(x)$ at an infinite potential barrier (impenetrable wall); without taking limits, all that can be concluded (on the basis of Hermiticity of the Hamiltonian) is

$$\cos\theta \ \psi(x) + \sin\theta \, d\psi(x)/dx = 0 \qquad (3)$$

with θ undetermined; only the familiar limiting process, in which increasingly high barriers are considered, leads to the correct boundary condition with $\theta = 0$.

The AB idealization is an example of the second sort, because however the limit is taken the result is always that of the original AB theory, based on (2). At least three limiting processes have been employed, corresponding to relaxation of the assumptions that the flux line is inaccessible, eternal and infinitely long. Kretschmar (1965) considered the cylinder containing the flux to be a potential barrier of finite height and therefore penetrable. Weisskopf (1961) considered the effects of the accessible electric field induced by slowly switching on the inaccessible flux. And Roy (1980) considered the effects of the accessible return flux from a solenoid of finite length. These analyses all lead to the same AB limit, in which there are observable effects of the flux, depending on α mod 1.

Any real physical system can at best approximate, but never attain, the idealizations of the original AB theory, so that these limiting processes correspond precisely to reality and explain the results of many experiments, all of which support AB theory. Returning to the analogy of the reflection boundary condition $\psi = 0$ (cf. the discussion surrounding on (3)), I would argue that it is wrong to claim this to be unphysical because no real potential barrier is infinite; on the contrary, the finiteness of real barriers justifies the limiting process by which $\psi = 0$ is derived as a usable and useful approximation. The same reasoning applies to the AB effect; hence the title of this note.

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