

Fig. 1 Geometry of helically coiled optical fibre.

Interpreting the anholonomy of coiled light

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Circular birefringence of purely geometric origin was recently predicted¹ and observed² in helically coiled monomode optical fibres, and widely reported³⁻⁵ as a successful application to photons of a general theory^{6,7} for phase shifts in adiabatically transported quantum states. However, earlier similar observations⁸⁻¹⁰ had been interpreted not by quantum mechanics but simply as a classical anholonomy, namely parallel transport of the polarization¹¹. Indeed, because the magnitude of the effect is independent of the wavelength of the light as well as Planck's constant, it might seem that 'classical' here means that not only quantum but also wave effects can be neglected. Here, I argue that these experiments, and their discrete analogues, are most appropriately described at the level of classical electromagnetism; the parallel transport law can then be derived (rather than assumed⁸⁻¹¹) and nonadiabatic polarization changes calculated.

In the quantum description¹, photons in right- or left-circularly polarized light are assumed to be in the eigenstate of positive or negative helicity defined by the local tangent vector $t(s)$ of the fibre (Fig. 1). Because the input and output ends of the fibre are parallel, the eigenstate is transported round a closed loop in t space and thereby acquires the geometrical phase shift appropriate to spin one, namely⁶ minus or plus the solid angle Ω subtended by the loop at the origin of t space. These opposite phase shifts $\mp \Omega$ imply¹ that the direction of linear polarization will be rotated by Ω , and it is this gyrotropy that was observed^{2,8-10}.

Underlying this successful prediction there are, however, several uncertainties. In the absence of any obvious governing Hamiltonian it is not clear why the phase continuation rule of the general theory⁶ is applicable. Even if it is, the fact that the two helicity states are degenerate suggests that the application could equally be made to any superposition of them, yielding different results. (Relativistic arguments¹² do yield the phase continuation rule, and also uniquely select the helicity states, but are inapplicable within fibres.) Moreover, it is hard to see how this type of quantum description can provide estimates of the probability that the coiling will produce nonadiabatic transitions to the opposite helicity. In any case, the high photon flux in all the experiments so far carried out makes a quantum description unnecessary.

At the level of geometrical optics (shortwave limit) it is known^{13,14} that in a medium with smoothly-varying refractive index μ the electromagnetic field vectors are parallel-transported along a light ray. This result should, however, not be invoked to explain experiments with monomode fibres, because their fields cannot legitimately be described by ray optics.

In classical electromagnetism the field is governed by Maxwell's equations, with μ depending only on perpendicular distance ρ from the fibre axis. Using the ideas of coupled local modes and the weak-guidance approximation¹⁵, we can write the transverse electric field E at position s along the fibre as a superposition of fields linearly polarized along the fibre normal $n(s)$ and binormal $b(s)$ ¹⁶; this is an adiabatic approximation (with s playing the role that time does in quantum mechanics), valid for gently coiled fibres. Thus,

$$E(\rho, s) = \exp\{i\beta s\} f(\rho) [c_1(s)n(s) + c_2(s)b(s)] \quad (1)$$

where β and $f(\rho)$ are, respectively, the propagation constant and modal amplitude appropriate to the straight fibre. Substitution into Maxwell's equation gives, after some analysis (to be published elsewhere), the following effective Hamiltonian evolution equation for the polarization coefficients c_1 and c_2 in terms of the fibre curvature κ and torsion τ :

$$i \frac{\partial}{\partial s} \begin{pmatrix} c_1(s) \\ c_2(s) \end{pmatrix} = \begin{pmatrix} \kappa^2(s)/2\beta & i\tau(s) \\ -i\tau(s) & 0 \end{pmatrix} \begin{pmatrix} c_1(s) \\ c_2(s) \end{pmatrix} \quad (2)$$

This simplest possible theory neglects radiative leaking, coupling to reflected modes and all elasto-optic effects.

Now $\beta \approx 2\pi/\lambda$ where λ is the light wavelength in the fibre cladding, and κ^{-1} and τ^{-1} are both comparable with the fibre bend distances, so in lowest approximation the term $\kappa^2/2\beta$ is negligible and the torsion terms dominate. Then, for initial linear polarization in direction α (relative to n), (2) gives

$$c_2(s)/c_1(s) = \tan \left(\alpha - \int_{-\infty}^s ds' (\tau(s')) \right) \quad (3)$$

This is precisely the parallel transport law for E , giving, after one complete helical turn, a polarization rotation equal to the

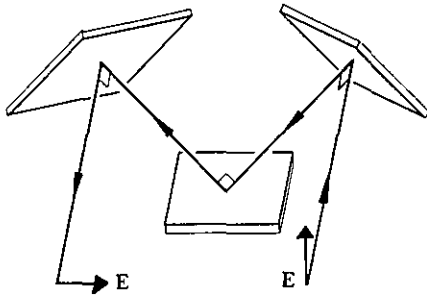


Fig. 2 Antiadiabatic rotation of polarization E by three reflections in metal mirrors.

solid angle¹⁰

$$\Omega = 2\pi - \int_{-\infty}^{\infty} ds' \tau(s')$$

Moreover, (2) resolves the degeneracy in favour of the helicity states: local eigenmodes (again neglecting $\kappa^2/2\beta$) are circularly polarized, that is, $c_2/c_1 = \pm i$, and acquire opposite phase shifts $\mp\Omega$ round the turn.

Nonadiabatic transitions are induced by the curvature term $\kappa^2/2\beta$. In lowest order this gives the probability for a change from (say) + to -, as

$$P_{\mp} = \left| \int_{-\infty}^{\infty} ds \kappa^2(s) \exp \left\{ 2i \int_{-\infty}^s ds' \tau(s') \right\} \right|^2 / 16\beta^2 \quad (4)$$

For a helix uniformly wound on a cylinder of radius R so as to produce phase shifts $\mp\Omega$, this can be written as

$$P_{+-} = [\Omega/2\pi(2 - \Omega/2\pi)]^3 \sin^2 \Omega / (1 - \Omega/2\pi)^2 / 16\beta^2 R^2 \quad (5)$$

Even for $R \sim 1$ mm this never exceeds 10^{-8} . For planar bends ($\tau = 0$) there is no polarization rotation, and the eigenmodes of (2) are linearly polarized with a tiny bend-induced shift $\kappa^2/2\beta$ in the local propagation constant of the mode polarized along n .

It has been suggested (J. N. Ross, personal communication and ref. 17) that the coiling of t can be accomplished by (at least three) discrete reflections, the resulting sudden changes in t being simulations of adiabatic change. But ideal mirrors (infinite conductivity) do not conserve helicity; they reverse it, and with this 'antiadiabaticity' the solid angle Ω must be accumulated with t replaced by $-t$ on alternate segments of the light path¹⁷. If the light beam is reversed after three successive reflections, each changing its direction by 90° (Fig. 2), the predicted polarization rotation (see also ref. 18, pages 84-86) is also 90° . Real metal mirrors (finite conductivity) do not quite reverse helicity: the 'nonadiabatic' probability for preserving the original helicity is

$$P_{++} \approx 1/(2|\mu|^2) \quad (6)$$

which for silver with $\mu = 0.2 + 3.44i$ is 0.042.

The simulation of true adiabatic change with discrete reflections can be achieved only by total internal reflection in a

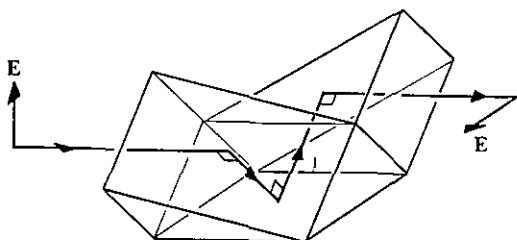


Fig. 3 Adiabatic rotation of polarization E by three internal reflections at the critical angle.

dielectric with index μ at the critical incidence angle $i = i_c = \sin^{-1}(\mu^{-1})$ (this follows from Fresnel's formulae¹³). For glass, $i_c \approx 45^\circ$, so that rotation of 90° can be accomplished by successive reflections in three 45° right prisms arranged as in Fig. 3. However, i_c is not precisely 45° (because $\mu \neq \sqrt{2}$), and this will cause nonadiabatic helicity switching, with probability

$$P_{+-} = (\mu^2 \sin^2 i - 1) / (\mu^2 \tan^2 i - 1) \quad (\sin i > \mu^{-1}) \quad (7)$$

For $i = 45^\circ$ and glass with $\mu = 1.5$, $P_{+-} = 0.1$.

The arrangements in Figs 2 and 3 form the basis of robust, simple and (nonadiabatic conditions notwithstanding) convincing lecture demonstrations of the anholonomic transport of polarization.

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