The nuts and bolts of fractals

The Science of Fractal Images
edited by Heinz-Otto Peitgen and Dietmar Saupe,
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Michael Berry

SEVEN YEARS after Benoit Mandelbrot published his definitive introductory essay on fractal geometry, few doubt its effectiveness and usefulness in the description of a tremendous variety of natural structures with many scales. Mandelbrot did not work out the details of applications; it was left to others to wield the tool he provided. Instead, the remarkable fractal images that adorned his book were drawn by computer.

Mandelbrot has been criticised for not providing the explicit recipes by which these pictures were created. I happen not to share this criticism, having discovered, in several cases, that digging hard enough into his terse Mathematical Addenda eventually yielded the information I wanted. But I have to admit that the presentation there is far from easy. What has been missing is a "how-to" book that explains at length and in fairly simple terms what has hitherto been available only in isolated magazine articles dealing with particular cases. That gap is what Heinz-Otto Peitgen and Dietmar Saupe set out to fill. In *The Science of Fractal Images*, they have collected the lecture notes of five courses given by themselves and three colleagues at a computer graphics meeting in 1987.

Richard Voss begins with a concise introduction to the fundamentals of fractal geometry. He defines the fractal dimension. This describes the self-similarity of an object: the fact that it can contain copies of itself ad infinitum. For fractals, the dimension need not be a whole number.

Voss emphasises the important difference between self-similar fractals and self-affine fractals. Self-similar fractals look the same (exactly or statistically) under the same magnifications in all directions. An example is a coastline. Self-affine fractals, on the other hand, look the same only under magnifications that are different in each direction. An example is the profile of a mountain range; gravity imposes a preferred direction on topographic forms.

Voss introduces self-affine fractal functions as generalisations of the graph of the position of a particle undergoing Brownian motion on a line, for which the dimension is 1·5. A fascinating and mysterious example is music, whose graphs (of pitch or loudness) have dimension 2 on scales from a tenth of a second to an hour, with apparently no dependence on culture from the Ba-Benzele Pygmies through folk songs of old Russia to the Beatles and Debussy.

Voss also introduces the common algorithms used to generate fractal landscapes and functions. My one criticism is of the example he uses to illustrate the idea that a fractal object can have a density which may be uniformly or irregularly distributed. In the latter case, the object can be a multifractal, with an infinite set of fractal dimensions defined by powers of the density. The example is a coastline, for which the range of dimensions is unconvincingly narrow.

Dietmar Saupe stresses the fact that a fractal description is a compact encoding of the structure of a complicated object, because "... the complexity of a fractal, measured in terms of the shortest computer program that can generate it, is very small".

AP

Paul Potter

The songs of pygmies and the Beatles are indistinguishable to a mathematician. When loudness or pitch is plotted against time, the fractal dimension always comes out as 2

His main contribution is the careful description of a series of methods for calculating fractal functions and landscapes. Like most of the others in the book, the algorithms are presented in pseudocode, which is a sort of computer Esperanto, constructed by amalgamating features of Basic, Pascal, and so on. Nonspecialists might be surprised to learn that what takes up most time in the production of a fractal landscape is not the mathematical work of computing its height above a base plane but its "rendering", that is its presentation as a convincing picture by the diffuse and specular reflection of simulated lighting.

Robert Devaney explains how fractals arise as abstract constructions in simple mathematical models of dynamics, such as repeated quadratic equations in one and two dimensions. The fractals can arise as attractors, which are sets to which all orbits tend. It is a pity that he missed the chance to point out that, in his example (the Hénon map), the mathematical reason for attraction is simply that the map contracts areas, and that this models the dissipative effect of friction in physical systems. Sometimes, fractals can arise in the opposite way, as repellers, his example being the celebrated Julia sets obtained by quadratic repetition in the complex plane. Devaney's presentation is well paced until the final description of the Julia sets of transcendental functions, where the level increases.

Heinz-Otto Peitgen continues the dynamical theme by describing in detail several ways to calculate the celebrated Mandelbrot set. This inhabits the plane of all complex quadratic equations and is a "one page dictionary" of the Julia sets of the individual equations. Each of these results from "... the competition of several centres for the domination of the plane ... Usually, an unending filligreed entanglement and unceasing bargaining for even the smallest areas results." No wonder the Mandelbrot set, which synthesises infinitely many fractal pictures, has been called "the most complex object mathematics has ever 'seen'."

Michael Barnsley's contribution is as delightful as it is unfamiliar. He shows how to generate strikingly realistic reproductions of natural objects such as leaves, clouds and smoke plumes as fractal attractors not of a single repeated transformation but of a sequence of transformation chosen randomly from a finite set. He gets the transformations from the largest scales of the original object, by a very simple tracing procedure.

There are several shorter contributions. Mandelbrot himself provides two: a foreword about the history of computer graphics associated with fractals, and an appendix about some of the latest ideas for making fractal landscapes more realistic—for example, by correctly incorporating river networks. Dietmar Saupe's appendix describes a way of generating fractals as character strings interpreted as drawing instructions rather like Logo turtle graphics. Yuval Fisher's appendix is an account of his new and apparently very efficient way of drawing the Mandelbrot set as the limit of a set of discs inside (or outside) it.

I recommend *The Science of Fractal Images* not as an introduction to fractal geometry but to anybody with a serious intention to produce fractal images without the pain of rediscovering existing algorithms. They will need access to a computer with good graphics and—for the landscapes—a lot of memory. □

This week's reviewers

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