STUDIES OF SEMICLASSICAL SPECTRA: WHERE NEXT?

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ABSTRACT

The spectrum of a quantum Hamiltonian H is the set of its energy levels and eigenstates. By the correspondence principle, the spectrum must be related to the trajectories of the corresponding classical system if this exists, but semiclassical connections have been hard to find when the trajectories are chaotic. Recent progress[1-4] has been based on two ideas:

1 Semiclassical Scales

Different spectral phenomena show up on different scales, measured in terms of Planck's constant h. For energy, a fine semiclassical scale is the mean level spacing $h^N/(d\Omega/dE)$ where N is the number of freedoms and Ω the classical phase-space volume within energy E. A coarse scale is h/T where T is the period of the shortest classical orbit. For wavefunctions, a fine scale[6,7] is the de Broglie wavelength h/p(q) where p is the magnitude of a typical classical momentum contributing at position q. A coarse scale[4] is $\sqrt{(hM_{QP})}$ where M_{QP} is an element of the stability matrix describing deviations from a closed orbit.

2 Universality

Spectra of all classically chaotic systems are nontrivially the same (apart from some considerations of symmetry[8,9]) on fine scales. There is an analogy between the semiclassical limit $h\rightarrow 0$ and the

critical point of a thermodynamic system. For energy levels the universal 'critical behaviour' is in the statistics (distribution of spacings between neighbouring levels, fluctuations of numbers of levels in a given interval, etc), which are those of the eigenvalues of ensembles of random matrices[10,11]. For wavefunctions, the universality is in the short-range spatial correlations[6].

The theory that implements these ideas is based on the ε-smoothed spectral operator

$$\Delta_\varepsilon(E) = \delta_\varepsilon(E-H) = -\,\mathrm{Im}\,[\pi(E+i\varepsilon-H)]^{-1}, \qquad (1)$$

whose trace, configuration-space matrix elements and Weyl transform are respectively the smoothed level density and the average over wavefunction products and Wigner functions of states within ε of E. These quantities can be expanded[12,13,4] as a smooth contribution from the energy surface H=E and oscillatory corrections from the closed orbits with energy E. Universality of the spectral statistics is a cooperative effect[1] of the long closed orbits, for which there is a corresponding classical universality[14] (in the case of the Riemann zeros[1,15] this is the prime number theorem). Short closed orbits give universality-breaking correlations over many level spacings. For the wavefunctions the short orbits contribute scars[3,4] which again give nonuniversal structure on larger scales, this time spatial.

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