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Quantum Chaology, Not Quantum Chaos

Michael Berry

H. H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, U.K.

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Abstract

There is no quantum chaos, in the sense of exponential sensitivity to initial conditions, but there are several novel quantum phenomena which reflect the presence of classical chaos. The study of these phenomena is quantum chaology.

In the 1970's it began to be widely appreciated that systems can behave unpredictably even when their evolution is causally determined by known dynamical laws [1]. The early applications were to classical mechanics [2], but it was natural for people to try to find chaos in the time development of quantum systems, in particular those for which the corresponding classical system is chaotic. Such attempts failed: it was found [3] that after a long enough time the chaos of classical mechanics is always suppressed by quantum mechnics. There is still no general analytical theory of this suppression, but there are several qualitative and semiquantitative explanations (such as the finite volume of the Planck cell in phase space [4–6], or the discreteness of energy or quasienergy spectra [7, 8]). It begins to seem that the phenomenon is fundamental, the ultimate reason for the absence of chaos being the linearity of Schrodinger's equation [9].

Although there is no quantum chaos, there have been several conferences devoted to it, at which someone unfailingly points to the paradox of a nonexistent subject inspiring the discovery of new aspects of quantum mechanics. In addition to the quantum suppression of chaos, these discoveries include random-matrix behaviour in the statistics of energy levels [10–13], quantization around classically ephemeral structures ("vague tori") in chaotic regions of phase space [14], and a rich variety of eigenfunction morphologies [15, 16].

My intention here is to dissolve the paradox by proposing a definition which captures the essence of what is actually being studied. I will call this *quantum chaology*. Then I will explain some of the key components of the definition.

Definition. Quantum chaology is the study of semiclassical, but nonclassical, phenomena characteristic of systems whose classical counterparts exhibit chaos.

"Chaology" revives a word [18] which two centuries ago was a technical term describing the branch of theology devoted to what existed before The Creation. I suggest that nowadays we should use it unadorned to mean the study of unpredictable behaviour in deterministic systems, and in the combination "quantum chaology" to denote the subject defined above.

"Semiclassical" means "as Planck's constant h tends to zero". The limit is nontrivial because quantum mechanics, considered as depending on a complex parameter h, is essentially singular at the "classical" origin $h = 0$, in ways that differ from system to system [19, 20]. This nonanalyticity is present in all waves in the limit of vanishing wavelength. The best understood nonanalyticities are associated with caustics in integrable systems, and can be expressed in terms of scaling laws involving exponents whose determination involves catastrophe theory [21]. Because of the essential singularity at $h = 0$, the classical limit of quantum mechanics (and also the geometrical-optics limit of electromagnetism) is complicated and conceals a rich variety of phenomena. Quantum theory is a nonperturbative extension of classical mechanics (unlike, say, special relativity, which grows out of Newtonian mechanics by a convergent perturbation expansion in velocity/c).

"Nonclassical" is incorporated into the definition to exclude the trivial sense in which classical unpredictability could be regarded as quantum chaos on the grounds that every classical system is really the $h = 0$ limit of a quantum one. In this way even fluid turbulence or the erratic orbits of some planetary satellites could masquerade as quantum chaos, but that would be a ridiculous use of the term.

"Characteristic of . . . chaos" is intended to exclude those semiclassical quantum phenomena that need have no relation to chaos, for example forbidden processes caused by barrier penetration.

With these interpretations, the definition does indeed correspond to what is being studied, as I now explain with examples.

The *suppression of chaos* [3] is a quantum phenomenon and therefore non-classical. It is of course characteristic of classically chaotic systems (for integrable systems there is no chaos to suppress). Moreover it is a semiclassically emergent phenomenon: the "break time", beyond which classical diffusion ceases, increases boundlessly as h vanishes [5], and unless h is small the diffusion never gets started and so cannot be said to be suppressed.

The *distribution of energy levels in bound systems* [17] is a quantum phenomenon, because in classical mechanics energy is a continuous variable. In calculating spectral statistics (for example the level spacings distribution) many eigenvalues are needed; these have high quantum numbers and so are semiclassical. Moreover the magnification required to "unfold" the spectrum, making the mean level spacing units, is h^{-N}, where N is the number of freedoms, and this is semiclassically large. Further, the distributions thus found are indeed characteristic of classically chaotic systems (integrable ones have different spectral statistics [10]).

The *morphologies of wavefunctions* (for example in the space of coordinates) are quantum phenomena, because in the classical limit waves oscillate infinitely fast (in fact on the scale of the de Brogile wavelength which is proportional to h) and so wavefunctions do not exist. Classically regular and chaotic systems display very different morphologies [20, 22]. These are indistinguishable in the ground state but emerge more clearly as the number of oscillations increases – that is, in the semiclassical limit.

The spirit of the definition is not restrictive. Rather it is intended to reflect in a positive way what distinguishes quantum from classical chaology, namely seeking, discovering and explaining new phenomena which although semiclassically emergent are nevertheless fully quantum-mechanical.

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