Physica D 38 (1989) 29–31
North-Holland, Amsterdam

FALLING FRACTAL FLAKES

M.V. BERRY
H.H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK

The rate at which a D-dimensional cluster of N smoke spherules falls through air is calculated. The cluster may be large or small in comparison with the mean free path of air molecules. For example, a cluster with $N=1000$ falls ten times more slowly with $D=1.8$ than when compacted into a sphere with $D=3$. Fractality is therefore important. Its effect would be to lengthen the nuclear winter, and should be taken into account in future modelling.

To celebrate Benoit Mandelbrot's birthday, here is a simple estimate of the rate at which fractal particles fall through air. I did the calculation in 1984 as part of an assessment of how the nuclear winter would be affected by the fact that smoke can be fractal [1–3]; this was not taken into account in the original studies [4–7]. The most important effect of fractality is to alter the optics of smoke [8,9]. The essential point is that coagulation of smoke spherules into fractal clusters will not reduce the absorption of light in the way that coagulation into solid spheres would. This makes the nuclear winter colder, possibly by several degrees [10]. The other effect, to be discussed here, is that fractality would prolong the nuclear winter because fractal clusters fall much more slowly than solid spheres and so would remain aloft longer.

Simons [11,12] and Hess et al. [13] have since published similar results, but my old calculation was so simple that I think it still worth presenting in its original form.

Consider a fractal cluster of dimension D, formed by the aggregation of N spherules, each of radius a and density ρ. For smoke, $D\approx 1.8$ [3] and $a\approx 20$ nm [1]. We wish to calculate the speed $v(N, D)$ with which the cluster falls under gravity (acceleration g) through air with viscosity η and density ρ_a.

For smoke, v is always small enough for the frictional force F on the cluster to be linear, i.e.

$$F=\alpha v\,. \tag{1}$$

Equating this to gravity minus buouancy gives

$$v=\frac{4\pi N(\rho-\rho_a)ga^3}{3\alpha}\,, \tag{2}$$

so the problem reduces to finding α (or, what is equivalent, the diffusion constant kT/α [14]).

There are two limiting regimes, in which the cluster radius R (defined as the rms distance between spherules) is much larger or much smaller than the mean free path L of the air molecules. R is given by the fractal relation

$$R=a\,N^{1/D}\,. \tag{3}$$

Here I have omitted a dimensionless constant whose value is close to unity. At height h in the atmosphere, L is given by

$$L=L_0\exp(h/h_0)\,, \tag{4}$$

where for air $L_0\approx 60$ nm and $h_0\approx 8$ km.

If $R\gg L$, friction is caused by air flow round the cluster, and we can estimate α by assuming that the cluster entrains the air inside it and using Stokes' law:

$$\alpha\approx 6\pi R\eta \quad (R\gg L)\,. \tag{5}$$

If $R\ll L$, friction is caused by the impacts of individual air molecules on the spherules. These present a cross section $A\approx \pi a^2 N$ if $D<2$ (because the cluster is geometrically transparent) and $A\approx \pi R^2=\pi a^2 N^{2/D}$ if $D>2$ (because the cluster is geometrically opaque). Thus

Essays in honour of Benoit B. Mandelbrot
Fractals in Physics – A. Aharony and J. Feder (editors)

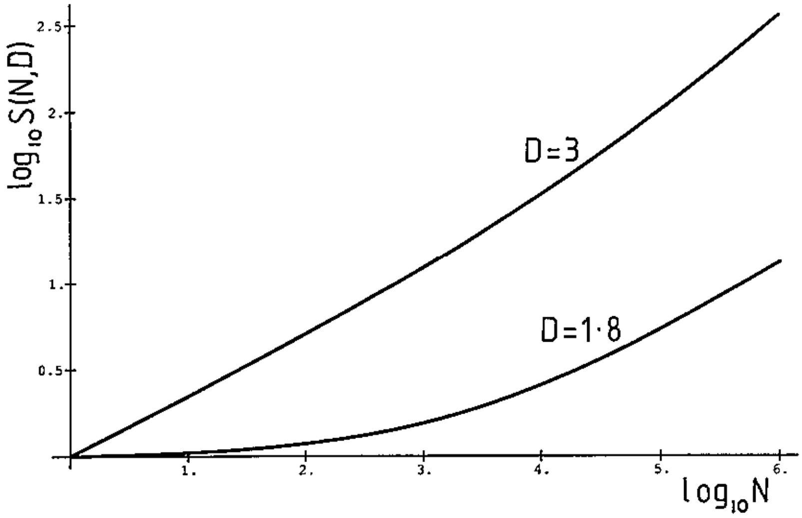

Fig. 1. Clustering speedup factor $S(N,D)$, calculated from eqs. (10) and (9).

$$A=\pi a^2 N^{\beta}, \quad \beta=1 \quad \text{if } D<2\,, \qquad \beta=2/D \quad \text{if } D>2\,. \tag{6}$$

If u is the average speed of air molecules we find from momentum balance that

$$\alpha \approx 2\rho_a u A \quad (R \ll L)\,. \tag{7}$$

We can eliminate $\rho_a u$ using $\eta=\rho_a uL/3$ [14], so that

$$\alpha \approx 6\pi a^2 N^{\beta} \eta / L \quad (R \ll L)\,. \tag{8}$$

When R/L is not very small or very large, the approximations underlying (5) and (8) do not justify anything more sophisticated than the simplest interpolation, which from (2) gives the speed of fall as

$$v(N,D)=\frac{2a^2\rho g N^{1-1/D}}{9\eta}\left(1+\frac{L}{aN^{\beta-1/D}}\right)\,. \tag{9}$$

For large clusters, the term involving L is negligible and $v \propto N^{1-1/D}$. When $D=3$ this gives the correct limit $N^{2/3}$. When $D=1$ it predicts v independent of N (so that, for example, the rate at which a hair falls would be unchanged if the hair were cut into pieces), which up to powers agrees with the known result $v \propto \log N$ [15]. For nuclear winter smoke, the mean free path can be large compared with R (because $h \approx 20$ km [7], cf. eq. (4)) and so its effects cannot always be neglected.

Fractality is important. For clusters with $N \approx 1000$, (9) predicts falling speeds of $v \approx 100$ m/yr for fractal clusters ($D=1.8$), as compared with $v \approx 1$ km/yr for solid clusters ($D=3$). A convenient dimensionless measure of how smoke falls faster as it coagulates is the clustering speedup factor

$$S(N,D) \equiv v(N,D)/v(1,D)\,. \tag{10}$$

This is shown in fig. 1 for $D=1.8$ and $D=3$. Obviously the cluster speeds up much more slowly if it is fractal. For example when $N=1000$ and $D=1.8$ (corresponding to a radius $R=930$ nm) $S=1.55$, whereas the same cluster with $D=3$ (i.e. solid, with $R=200$ nm) has $S=12.4$.

Of course it would be naive to infer from (9) that the fractality of smoke implies that the nuclear winter would last ten times longer than if smoke were not fractal. One reason is that the proportion of smoke that would coagulate dry, into fractals (as opposed to wet, into spheres) is the subject of controversy. Nevertheless, fractals fall so much more slowly than non-fractals that the effect of even a small proportion on the duration of a nuclear winter seems to be severe enough to warrant it being included in future modelling.

I thank Dr. S. Simons for telling me about his and other recent contributions to this subject.

References

[1] P. Chylek, V. Ramaswamy, R. Cheng and R.G. Pinnick, Appl. Opt. 20 (1981) 2980–2985.

[2] S.R. Forrest and T.A. Witten Jr., J. Phys. A 12 (1979) L109–L117.

[3] R.D. Mountain and G.W. Mullholland, in: Kinetics of Aggregation and Gelation, Proceedings of the International Topical Conference, F. Family and D.P. Landau, eds. (North-Holland, Amsterdam, 1984).

[4] P.J. Crutzen and J.W. Birks, AMBIO 11 (1982) 114–125.

[5] R.P. Turco, O.B. Toon, T.P. Ackerman, J.B. Pollack and C. Sagan, Science 222 (1983) 1283–1292.

[6] A.B. Pittock, T.P. Ackerman, P.J. Crutzen, M.C. Maccracken and C. Sagan, in: Environmental Consequences of Nuclear War (SCOPE 28), Vol. 1. Physical and Atmospheric Effects, F. Warner, ed. (Wiley, New York, 1985).

[7] O. Greene, I.C. Percival and I. Ridge, Nuclear Winter (Polity Press, Cambridge, 1985).

[8] M.V. Berry and I.C. Percival, Optica Acta 33 (1986) 577–591.

[9] J. Nelson, J. Mod. Opt. (1989), in press.

[10] J. Nelson, Nature (1989), in press.

[11] S. Simons, J. Phys. A 19 (1986) L901–L905.

[12] S. Simons, J. Phys. D 20 (1987) 1197–1199.

[13] W. Hess, H.L. Frisch and R. Klein, Z. Phys. B 64 (1986) 65–67.

[14] F. Reif, Fundamentals of Statistical and Thermal Physics (McGraw-Hill, New York, 1965).

[15] A. Berry and L. Swain, Proc. R. Soc. (London) 101 (1923) 766–778.