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REVIEW OF: DOES GOD PLAY DICE? THE MATHEMATICS OF CHAOS.

by Ian Stewart
(Basil Blackwell 1989, pp 317, hardback £15)

Many people who are not mathematically or technically minded are now aware that there is a new science devoted to the study of chaos. Popular accounts have appeared in newspapers and on television, and James Gleick's book ('Chaos: making a new science') was a best-seller. This excitement is justified, because chaology represents a revolutionary shift in our view of scientific explanation.

Previously, one of our main tests of our understanding of something was the ability to predict how it would happen. The paradigm was celestial mechanics, which (for example) enabled eclipses to be reliably predicted (and retrodicted). But this predictably clashes with the common experience that events in the world of everyday usually occur by chance. We cannot foretell whom we will marry, or how we will die. This type of unpredictability was explained as being the result of ignorance; most events are the outcome of many causes, and we cannot know them all. If we did, then since we know the laws (of physics) through which they act, predicting what will happen is a matter of calculation, given a big enough computer.

Nowadays, we know that this explanation is wrong. Some unpredictable behaviour is not the result of our ignorance of circumstances but is caused by instabilities inherent in the laws themselves. A system need not be complicated to be unpredictable. The random bouncing in a pinball machine is a direct consequence of Newton's laws of motion even in the idealized situation where there are no air currents, the ball is perfectly round and the table flat. This random outcome of deterministic laws is what scientists now call chaos.

It is philisophically profound, because it means that knowledge of the laws does not imply predictability of the behaviour. It is also increasingly practical. The root of our inability to forecast weather beyond a few days is not ignorance of the physics of air but chaos in the solutions of the equations of aerodynamics. Heart attacks seem to be caused by a transition to chaos in an oscillator whose healthy state is a regular pulsation.

Such a grand idea deserves to be widely known and so demands exposition at every level. Ian Stewart's account centres on the recognition (emphasized in his subtitle) that the essence of chaos is mathematical. This is because the laws of nature are expressed as equations, whose solutions embody behaviour. How can equations, compactly coded in a handful of symbols that describe the inevitable and unique manner in which the future follows from the past, predict randomness?

Stewart leads up to this paradox by explaining how alongside deterministic classical physics there arose the sciences of randomness - statistics and probability - whose development was motivated by the need for precise treatment of many individually unpredictable data in biological populations such as people. These sciences developed along almost parallel tracks, meeting only in the study of the average dynamics of large numbers of particles (such as gas molecules). Chaology began when Poincaré discovered (about a century ago) that randomness is inescapable even in the Newtonian motion of just three particles, mutually gravitating.

Stewart then shows how infinitely complicated and random evolution follows from a yet simpler stripped-down dynamics consisting of the repitition of a rule which, given one number, delivers another. He goes on to describe the explosion of modern research, and its many applications, fuelled not only by the proofs of theorems but also the availability of computer graphics as a powerful aid to mathematical discovery.

The explanations are clear and enlivened with friendly analogies, as we have come to expect from this author who is one of the few mathematicians who bother to try to explain their craft to a wider public. But I cannot say that this is a popular book. Those who are completely ignorant of mathematics will find it hard going. Attempts to make the mathematics palatable by using familiar language can backfire and mystify (as when the Farey series - a way of arranging fractions - is described as a 'gadget'). A real turn-off for the non-expert is Stewart's occasional use of formulae. It is a sad fact that in a society where uneducated people can follow the technicalities of knitting patterns or football-pool coupons we have to feel inhibited from presenting a logical argument in its natural language, that is in symbols. The alienation even afflicts those who are supposed to have higher education: the instructions to reviewers for this periodical forbid me to use formulae 'because of difficulties of typesetting'!

However, for those who have even rudimentary mathematical knowledge, for teachers and for lively-minded high school and university students, Ian Stewart gives a valuable insight into the innards of chaos.

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