QUANTUM THEORY NEAR THE CLASSICAL LIMIT

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Short-range correlations of the energy levels of highly excited quantum systems with no symmetry show universal statistics if the corresponding classical system has chaotic orbits. The universality has been demonstrated by several models in which time-reversal symmetry is broken. A theory predicting that at long range the spectral statistics are not universal is confirmed in a tantalising connection with the theory of prime numbers.

There is a great deal of interest in the quantum mechanics of systems whose classical orbits are chaotic. This activity is motivated by experiments on the high excited states of polyatomic molecules and atoms in strong fields and by the scandalous theoretical situation that after sixty years the relation between classical and quantum mechanics is still not fully understood.

One focus of research is the fine structure of the distribution of spectral energies. When the classical orbits are not chaotic the spectrum is well understood: if there are N freedoms, each quantum state is labelled by N quantum numbers and the levels obtained by varying just one quantum number form a locally equally-spaced sequence. No such labelling is possible when the orbits are chaotic, and the states must be indexed in order of energy by a single integer n (the ground state being n=1).

The lack of theoretical understanding in the chaotic case is indicated by the absence of an asymptotic quantum condition that would give the energy of the nth state with an accuracy that increases with n. But if attention is directed away from individual states and towards the statistics of their distribution, a remarkable universality emerges. All chaotic spectra (apart form some pathological exceptions) in each of three classes have identical short-range statistics. The three classes are bosonic and fermionic systems with time-reversal symmetry, and systems with no symmetry. Examples of short-range statistics are the probability distribution P(S) of the spacing S between nearest-neighbour levels, and the variance Σ(L) of the number of levels in an energy range where the expected number L is not too large.

Recent research at Bristol has concentrated on the most general case, where there is no symmetry. Then the universal spectral statistics are the same as those of the "Gaussian unitary ensemble" (GUE) of random complex Hermitian matrices. The theory is far from complete, and much progress has come from computer experiments on carefully-devised models. A simple way of destroying time reversal is to add a magnetic field to a "quantum billiard", that is a charged particle moving in a plane region whose boundary is a partly convex and partly concave specular reflector. It is convenient to concentrate the field into a single line of magnetic flux threading the region, thereby obtaining "Aharonov-Bohm chaotic quantum billiards". Using a technique of conformal transformation to solve the Schrödinger equation, several hundred levels can be obtained, sufficient to calculate spectral statistics. Both P(S) and Σ(L) conform to GUE predictions.

To test the generality of the GUE statistics, time reversal was broken in a different way, not involving magnetic fields. A spin-1/2 particle obeying the massless Dirac equation was trapped in a region whose walls reflect through a 4-

scaler repulsion. The computed spectra of these "neutrino billiards" also exhibit GUE statistics, showing that the theoretical ideas are strong enough to survive into relativistic chaos.

It is beginning to be understood that quantum spectral universality has its origin in a classical universality of the distribution of very long closed orbits. The same theory predicts that the geometry of the short closed orbits will affect the spectral statistics at long range and these will therefore not be universal. In particular, $\Sigma(L)$ should deviate from the GUE prediction for large L. Testing this result requires thousands of levels and so is difficult. Recent computations of the spectrum of hydrogen in a strong magnetic field have shown the beginnings of the departure from universality.

A full and detailed confirmation of the theory has however come from a surprising source: the zeros of the Riemann zeta function of prime number theory. The celebrated Riemann hypothesis states that these zeros lie on a certain line in the complex plane. Analysis suggests that they are distributed on the line like the energy levels of some (unknown) quantum system without time reversal symmetry whose classical counterpart is chaotic. Very many zeros have been computed (10^5 near the 10^{12}th) and provide a laboratory for testing semiclassical theories. Thus quantum mechanics and chaos seem to be profoundly connected with one of the most difficult problems of arithmetic.

The Quantum Chaology Group in the Physics Department at Bristol University is led by Professor M V Berry. It was established in 1976 and now comprises two permanent staff, two RAs and three PhD students. Its areas of interests are quantum reflections of classical chaos, phase in quantum mechanics, and semiclassical asymptotics. The Group collaborates with the Weizmann Institute, Rehovot, Israel.