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Berry M V. Uniform approximation for potential scattering involving a rainbow.
Proc. Phys. Soc. London 89:479-90, 1966.
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This was the first paper in a series about quantum scattering near the classical limit. Approximations previously restricted to the neighbourhood of foci of classical orbits were extended so as also to be valid far from these singularities. [The *SCI*® indicates that this paper has been cited in over 115 publications.]

Quantum Asymptotics of Rainbows

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My work emerged naturally from the confluence of four streams. First was the experience and mastery of mathematical asymptotics (later distilled into a magnificent text[1]) of my PhD supervisor, R.B. Dingle. Second was my attending, by chance, an inspiring series of lectures by J.L. Synge on the Hamiltonian theory of systems of rays as analogues of wavefields.[2] Thus I arrived in Bristol in 1965, determined to fuse these approaches into a complete multidimensional extension of the WKB method, to yield a technique for getting approximations to quantum mechanics in the semiclassical limit (small Planck's constant). (This aim has still not been achieved in full generality.)

A natural physical problem containing some of the difficulties of the general case soon presented itself: scattering of a beam of particles by a central field of force. This had been studied experimentally and theoretically in several contexts and especially by chemists wanting to determine interatomic and intermolecular potentials. Their papers led me to the seminal works of K.W. Ford and J.A. Wheeler,[3,4] published in 1959, which was the third of my streams. They discovered that scattering cross-sections in the semiclassical limit are dominated by focal singularities of the family of parallel trajectories incident on the scatterer. Of several effects they studied, the most dramatic was the quantum rainbow. This is associated with an extremum of the classical deflection angle as a function of impact parameter. Close to the rainbow angle, Ford and Wheeler identified characteristic quantum oscillations caused by the interference of the two contributing classical paths. The oscillations were described by the same function that G.B. Airy had introduced in 1838 to describe the analogous effect in the optical rainbow.[5] Their approximation captured the essence of the semiclassical singularity but failed to match the known nonsingular path contributions far from the rainbow. In the language of asymptotics, theirs was a transitional approximation, not uniformly valid in angle.

The fourth stream was the method for obtaining the desired approximation, published by C. Chester, B. Friedman, and F. Ursell[6] in 1957. They showed how to generalize the method of stationary phase for integrals so as to be uniformly valid when two stationary points coalesce as a parameter varies. My paper applied their technique to quantum scattering, identifying the integral as the Poisson-approximated sum over partial-wave angular momenta, the parameter as the observation (deflection) angle, and the coalescing stationary points as the contributing rays. The final formula was numerically very accurate[7] and also simple in structure: The uniform approximation to the scattering amplitude is the sum of an Airy function and its derivative, with arguments and prefactors depending only on the actions of the two contributing classical paths (even when these are complex).

Because of this simplicity and accuracy, the result has been applied to the inversion of experimental scattering data.[8] Uniform approximations—including those of more complicated type, involving the coalescence of more than two trajectories[9,10]—are now applied routinely in scattering theory.

1. Dingle R B. *Asymptotic expansions: their derivation and interpretation.* New York: Academic Press, 1973. 521 p. (Cited 155 times.)
2. Synge J L. The Hamiltonian method and its application to water waves. *Proc. Roy. Irish Acad. Sect. A* 63:1-34, 1963. (Cited 5 times.)
3. Ford K W & Wheeler J A. Semiclassical description of scattering. *Ann. Phys. NY* 7:259-86, 1959. (Cited 510 times.)
4. ————. Application of semiclassical scattering analysis. *Ann. Phys. NY* 7:287-322, 1959. (Cited 180 times.)
5. Airy G B. On the intensity of light in the neighbourhood of a caustic. *Trans. Camb. Phil. Soc.* 6:379-402, 1838. (Cited 50 times since 1945.)
6. Chester C, Friedman B & Ursell F. An extension of the method of steepest descents. *Proc. Camb. Phil. Soc.* 53:599-611, 1957. (Cited 150 times.)
7. Mullen J M & Thomas B S. On an evaluation of the accuracy of the uniform semiclassical approximation for differential elastic scattering cross sections. *J. Chem. Phys.* 58:5216-21, 1973. (Cited 25 times.)
8. Buck U. The inversion of molecular scattering data. *Rev. Mod. Phys.* 46:369-89, 1974. (Cited 105 times.)
9. Berry M V. Waves and Thom's theorem. *Advan. Phys.* 25:1-26, 1976. (Cited 135 times.) [See also: Berry M V. Catastrophes and waves. Citation Classic. *Current Contents/Engineering, Technology & Applied Sciences* and *CC/Physical, Chemical & Earth Sciences* 7 May 1990.]
10. Connor J N L & Farrelly D. Theory of cusped rainbows in elastic scattering: uniform semiclassical calculations using Pearcey's integral. *J. Chem. Phys.* 75:2831-46, 1981. (Cited 20 times.)