

## Quantal reflections of classical chaos

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In the semiclassical limit, that is as Planck's constant  $\hbar \rightarrow 0$ , quantum energy levels display remarkable universality. This is seen by magnifying the spectrum so that the levels have unit mean density (the magnification is of order  $\hbar^{-N}$  where  $N$  is the number of freedoms), and studying the statistics of the numbers thus obtained. There are four universality classes for the level fluctuation statistics, depending on whether the classical trajectories are integrable or chaotic and, if chaotic, whether the system has time-reversal symmetry and, if it does, whether it is bosonic or fermionic. An example of numbers in the most general class (chaotic, no symmetry) are the imaginary parts of the Riemann zeros. The universality classes apply when the dynamics is purely chaotic or purely integrable (i.e. not mixed). Even then the universality is restricted to energy ranges of order  $\hbar$ , i.e. correlations among a number of order  $\hbar^{-(N-1)}$  levels. These 'experimental' facts can be explained in essence and, for some statistics, in detail, by representing the quantum spectral fluctuations as a sum over classical periodic orbits. Quantum universality stems from universality in the distribution of long classical orbits. Difficulty in obtaining a quantization formula for the chaotic case arises from the fact that the sum over classical orbits diverges for real energy. A conjectured resummation (analytic continuation), giving a finite quantization rule, can be written down by analogy with the Riemann-Siegel formula of number theory.

### References

- M.V. Berry, 'Quantum chaology' Proc. Roy. Soc. Lond. A413 (1987) 183-198.
- M.V. Berry, 'Some quantum-to-classical asymptotics' Les Houches Lecture Series 52 (eds: M-J Giannoni and A. Voros) to be published by North-Holland 1990.
- M.V. Berry and J.P. Keating, 'A rule for quantizing chaos?' submitted to J. Phys. A 1990.