LECTURES ON MECHANICS

By J. E. Marsden: 254 pp., £19.95, ISBN 0 521 42844 0
(Cambridge University Press, 1992).

Mechanics is in a period of intense development. New concepts are being developed, and old ideas substantially generalised. These lectures, given between 1989 and 1991, reflect this activity, by presenting in detail several topics close to the author's research interests.

A central theme is Hamiltonian reduction. This is the judicious use of symmetry to project a high-dimensional dynamical system onto a subspace on which the dynamics remains Hamiltonian. A good example (one of several given here, as well as the general theory) is a free rigid body, which has 6 freedoms and hence a 12-dimensional phase space. By transforming away the translations, and transferring to a body frame, motion can be reduced to a 2-sphere, which is the phase space of a system with just one freedom (the 'coordinate' is the azimuth angle, and the 'momentum' is the cosine of the polar angle).

There is a substantial treatment of geometric angles, the first in any book, to my knowledge. These are shifts in the orientation of one part of a dynamical system when another part is driven through a cycle. A good example is the Foucault pendulum, where the Earth's rotation drives the symmetry axis (the vertical) round a cone, whose solid angle gives the geometric angle by which the direction of swing turns relative to an inertial frame (this differs by 2π from the turn seen on the rotating Earth, a point glossed over here). Several potentially useful 'holonomy drives' (a nice term) are described, including that actually used by a falling cat to change its orientation while its angular momentum remains zero. Some irritating features of this treatment are: use of the term 'geometric phases' to denote these geometric angles (this invites confusion with the phases of quantum mechanics, which must be differentiated with respect to actions to obtain the classical angles); not presenting Hannay's beautiful general theory for the geometric angles in integrable systems, in terms of the cycled invariant tori; not giving proper credit to Shapere, Wilczek and Nityananda for the attitude shift of inertial systems whose shape is cycled (falling cats).

With so much exploration of the content of dynamical equations being carried out by numerical computation, it is useful to have a chapter on the theory of mechanical integrators, that is, solution schemes for (mainly Hamiltonian) equations. Numerical approximations should conserve constants of motion (for example, energy), and preserve the Hamiltonian (symplectic) structure. It is shown here that these desiderata cannot be satisfied simultaneously.

There is a chapter on control mechanisms to stabilise potentially unstable motion using feedback. Several interesting examples are given. I wonder if the proposed schemes are already well-known to engineers or are really new, and what their relation is to the methods being developed by Grebogi, Ott and Yorke to control chaos by stabilising unstable periodic orbits.

When parameters are varied, dynamics can change qualitatively, in ways whose classification is the province of bifurcation theory. The treatment given here for Hamiltonian systems is fairly conventional, apart from an account of some recent work by the author of the splitting of separatrices in perturbed integrable systems, which opens up exponentially narrow zones of chaotic motion.

Bad English mars the presentation; readers should not have to suffer 'this phenomena' and 'with respect of'. Nevertheless, I recommend this book for purchase by mathematics libraries, as a source of useful references and worthwhile modern material not easily accessible elsewhere.

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INDEX OF BOOK REVIEWS