THREE COMMENTS ON THE AHARONOV-BOHM EFFECT*

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The discovery by Aharonov and Bohm in 1959 (and its partial anticipation ten years earlier by Ehrenberg and Siday) was surprising and important and has been a fertile source of further discoveries. Neverthless some aspects of it have since become the subject of unnecessary mystification, embodied in three commonly held views on which I will comment.

1. THE AB EFFECT CONTRADICTS COMMON SENSE.

The 'common sense' is that fields are physical entities, whereas the potentials the AB effect relies upon (to describe nonlocal influences of fields) are mere mathematical conveniences. This opinion is ahistorical because it ignores the fact that fields were themselves originally conceived as mathematical conveniences - to help describe the distant (that is nonlocal) effects of charges and gravitating masses. These actions at a distance were certainly not common sense, and children who play with magnets or are told that the tides are caused by the moon find these phenomena miraculous today (so do I). One of AB's achievements was to contribute to the process whereby yesterday's mathematical constructions prove their worth by pointing to new phenomena, and so become regarded as physical entities today.

2. THE AB EFFECT IS NONCLASSICAL.

This is doubly misleading. First, although distant fields play no part in Newtonian classical mechanics (which deals only with forces), they contribute in Hamiltonian classical mechanics via the action (which depends on canonical rather than kinetic momentum and so involves the vector potential). Second - and in a deeper sense - the effect is a general one of wave physics, not confined to quantum mechanics. Any wave on a current, that is in a medium moving with velocity $\boldsymbol{u}(\boldsymbol{r})$ (much smaller than the wave speed), satisfies an equation with the same structure as Schrödinger's, in which $\boldsymbol{u}(\boldsymbol{r})$ plays the role of a vector potential. It follows that the analogue of a vector potential which describes a field vanishing everywhere outside a line is a flow for which $\nabla\times\boldsymbol{u}(\boldsymbol{r})=0$ outside the line, and this is the flow of an irrotational vortex. Therefore one way to display the AB wavefunction is to let ripples on the surface of water encounter a bathtub vortex. (The analogue of the dimensionless AB quantum flux parameter is the circulation of the vortex, divided by the product of the

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* Following the lectures by A. Tonomura (this volume).

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ripples' wavelength and group velocity.) This experiment was successfully carried out (Berry et al 1980). Taking a wider perspective, there are four sorts of AB effect, depending on whether the wave or the flux is quantum or classical. In AB's original proposal, and most subsequent experiments, the wave was quantum and the flux classical. In the more recent experiments by Tonomura, involving superconducting rings, both the wave and flux were quantum. V. Steinberg (private communication) is exploring the 'classical-quantum' case, in which waves in liquid helium would encounter rotons (quantized vortices). And of course the bathtub experiment described above is the 'classical-classical' case.

3. THE AB EFFECT DEPENDS ON IDEALISATIONS THAT DO NOT APPLY IN REALITY.

The idealisations were that the electron wave does not penetrate into the magnetic flux, that the solenoid is infinitely long (so that there is no return flux) and that the flux has been switched on forever (so that there was never any electric field induced outside by the changing flux). In any real experiment these ideal conditions are achieved only approximately (even in Tonomura's experiments with superconducting rings there must be some penetration of the flux by the electrons - of order 10^{-23}, or perhaps even $\exp\{-10^{+23}\}$, but strictly nonzero). But when there is the slightest violation of the idealisations it is possible (as several people have shown) to choose a formulation in which the fringe shift arises entirely from the tiny real fields that coexist with the electrons. Therefore there is a sense in which the AB effect - nonlocal influences of distant fields - does not exist in reality. However, there is a heavy price for maintaining this view. As the idealisation-violating fields get smaller, the fringe shifts approach that calculated by AB. Therefore those who maintain that the shifts arise from real fields (as opposed to the gauge-invariant phase factor involving the vector potential) must tolerate finite causes produced by infinitesimal effects. Elsewhere (Berry 1986), I have discussed this point, and its relation to other limiting processes in physics, in a little more detail.

References

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