SPECTRAL TWINKLING: A NEW EXAMPLE OF SINGULARITY-
DOMINATED STRONG FLUCTUATIONS (summary)

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The energy levels of quantum systems whose classical counterparts exhibit mixed
chaology obey neither random-matrix nor Poisson statistics. J.P. Keating, H. Schomerus
and I have recently suggested [1] that there are universal aspects of these quantum spectral
statistics that are characteristic of mixed systems, associated with the bifurcations of stable
and unstable isolated periodic orbits that distinguish the corresponding classical mechanics
from the purely chaotic and purely integrable cases. We study the Planck \((\hbar)\) dependence
of the moments \(M_m(\hbar)\) of the fluctuating part of the level-counting function (spectral
staircase), and argue that these diverge in the classical limit, according to the scaling law
\(M_m(\hbar)\sim \hbar^{-\nu(m)}\). To determine the ‘twinkling exponents’ \(\nu(m)\), the spectral staircase is first
represented as a sum over classical periodic orbits [2], corrected to eliminate the
divergences near bifurcations [3, 4]. Then, using the \(\hbar\)-scaling appropriate to each
bifurcation, its contribution to \(M_m(\hbar)\) is estimated. The exponents \(\nu(m)\) then follow from a
competition over all bifurcations (with different values of repetition number and
codimension). This ‘battle of the bifurcations’ depends on new results about the hierarchy
of their associated normal forms.

There are several other examples of statistics falling into this class [5], where large
fluctuations are dominated by geometrical singularities. In twinkling starlight [6, 7], the
singularities are caustics, of light focused by atmospheric turbulence, which dominate the
wavelength-dependence of the light intensity, and the classification of caustics required for
their competition is catastrophe theory [8, 9]. The density of states \(n(E)\) of a solid, in the
case where there are many van Hove singularities, also requires the catastrophe
classification, and the quantity to be calculated – again the result of a competition – is the
universal power-law decay of the tail of the probability distribution of values of \(n(E)\) [10].
Finally, the sex life of moths is dominated by the male’s search for the female by smelling
the ‘odour plume’ that she emits [11]. In a turbulent wind, this is determined by the
fluctuating concentration \(C\) of a passive scalar (pheromone) that is convected by the flow
while diffusing, with diffusion constant \(D\). The small-\(D\) asymptotics of the moments of \(C\)
are determined by the streak line singularity, consisting of all those fluid particles that have
passed through the female [5, 12]. In all these cases, the fluctuations are governed by power laws originating in geometric singularities that have no connection with fractals.

REFERENCES


