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Spectral Twinkling: A New Example of Singularity-Dominated Strong Fluctuations (summary)

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Abstract

In quantum systems that are neither completely integrable nor completely chaotic, energy-level fluctuations are governed by asymptotic scaling laws, analogous to those for the intensity fluctuations of twinkling starlight.

The energy levels of quantum systems whose classical counterparts exhibit mixed chaology obey neither random-matrix nor Poisson statistics. J. P. Keating, H. Schomerus and I have recently suggested [1] that there are universal aspects of these quantum spectral statistics that are characteristic of mixed systems, associated with the *bifurcations* of stable and unstable isolated periodic orbits that distinguish the corresponding classical mechanics from the purely chaotic and purely integrable cases. We study the Planck (h) dependence of the moments $M_m(h)$ of the fluctuating part of the level-counting function (spectral staircase), and argue that these diverge in the classical limit, according to the scaling law $M_m(h) \sim h^{-\nu(m)}$. To determine the "twinkling exponents" $\nu(m)$, the spectral staircase is first represented as a sum over classical periodic orbits [2], corrected to eliminate the divergences near bifurcations [3,4]. Then, using the h-scaling appropriate to each bifurcation, its contribution to $M_m(h)$ is estimated. The exponents $\nu(m)$ then follow from a competition over all bifurcations (with different values of repetition number and codimension). This "battle of the bifurcations" depends on new results about the hierarchy of their associated normal forms.

There are several other examples of statistics falling into this class [5], where large fluctuations are dominated by geometrical singularities. In twinkling starlight [6,7], the singularities are *caustics*, of light focused by atmospheric turbulence, which dominate the wavelength-dependence of the light intensity, and the classification of caustics required for their competition is catastrophe theory [8,9]. The density of states $n(E)$ of a solid, in the case where there are many *van Hove* singularities, also requires the catastrophe classification, and the quantity to be calculated again the result of a competition is the universal power-law decay of the tail of the probability distribution of values of $n(E)$ [10]. Finally, the sex life of moths is dominated by the male's search for the female by smelling the "odour plume" that she emits [11]. In a turbulent wind, this is determined by the fluctuating concentration C of a passive scalar (pheromone) that is convected by the flow while diffusing, with diffusion constant D. The small-D asymptotics of the moments of C are determined by the *streak line* singularity, consisting of all those fluid particles that have passed through the female [5,12]. In all these cases, the fluctuations are governed by power laws originating in geometric singularities that have no connection with fractals.

References

1. Berry, M. V., Keating, J. P. and Schomerus, H., Universal twinkling exponents for spectral fluctuations associated with mixed chaology, Proc. Roy. Soc. Lond., **A456**, 1659 (2000).
2. Gutzwiller, M. C., Periodic orbits and classical quantization conditions *J. Math. Phys.* **12**, 343 (1971).
3. Ozorio de Almeida, A. M. and Hannay, J. H., Resonant periodic orbits and the semiclassical energy spectrum J. Phys. A. **20**, 5873 (1987).
4. Sieber, M., Uniform approximation for bifurcations of periodic orbits with high repetition numbers J. Phys. A. **29**, 4715 (1996).
5. Berry, M. V., Spectral twinkling, in Enrico Fermi School "New Directions in Quantum Chaos", (ed. U. Smilansky), (Italian Physical Society, Bologna, Course CXLIII, 45 (2000)).
6. Berry, M. V., Focusing and twinkling: critical exponents from catastrophes in non-Gaussian random short waves J. Phys. A **10**, 2061 (1997).
7. Walker, J. G., Berry, M. V. and Upstill, C., Measurement of twinkling exponents of light focused by randomly rippling water Optica Acta **30**, 1001 (1983).
8. Arnold, V. I., Catastrophe Theory (Springer, Berlin, 1986).
9. Poston, T. and Stewart, I., "Catastrophe theory and its applications", (Pitman (reprinted by Dover), London, 1978).
10. Berry, M. V., Universal power-law tails for singularity-dominated strong fluctuations J. Phys. A **15**, 2735 (1982).
11. Murlis, J. and Jones, C. D., Fine-scale structure of odour plumes in relation to insect orientation to distant pheromone and other attractant sources Physiol. Entomol. **6**, 71 (1981).
12. Berry, M. V., Ipotesi di scala e fluttuazioni non gaussiane nella teoria catastrofica della onde (Italian translation of "Scaling and nongaussian fluctuations in the catastrophe theory of waves") in "Prometheus", (eds. P. Bisogno and A. Forti), 1985, Vol. 1, pp. 41–79.