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Foreword

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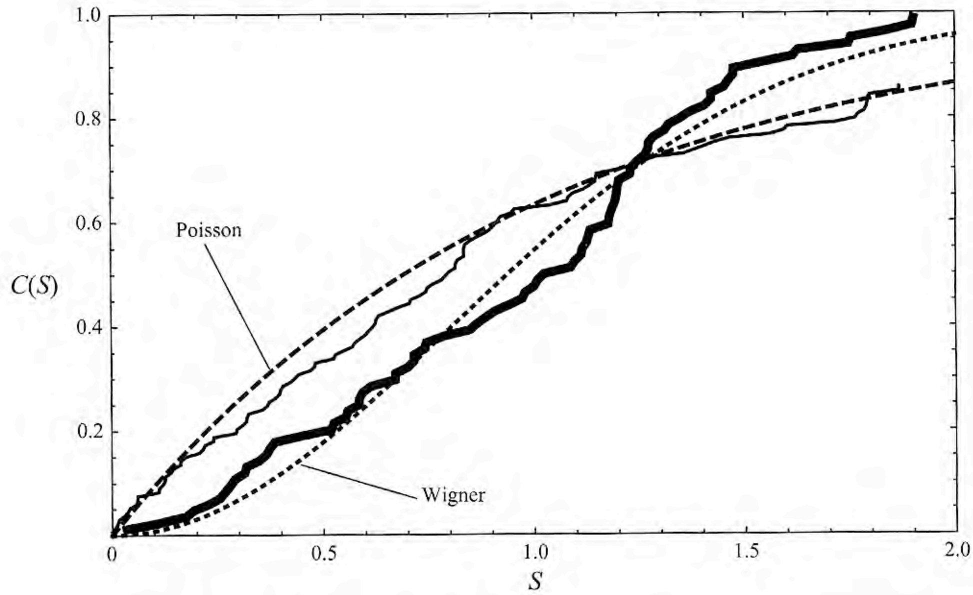
In the early 1970s, Martin Gutzwiller and Roger Balian and Claude Bloch described quantum spectra in terms of classical periodic orbits, and in the mid 1970s it became clear that the random matrix theory devised for nuclear physics would also describe the statistics of quantum energy levels in classically chaotic systems. It seemed obvious even then that these two great ideas would find application in acoustics, but it has taken more than three decades for this insight to be fully implemented. The chapters in this fine collection provide abundant demonstration of the continuing fertility, in the understanding of acoustic spectra, of periodic orbit theory and the statistical approach. The editors' kind invitation to me to write this foreword provides an opportunity to make a remark about each of these two themes.

First, here is a simple argument for periodic orbit theory being the uniquely appropriate tool for describing the acoustics of rooms. The reason for confining music and speech within auditoriums – at least in climates where there is no need to protect listeners from the weather – is to prevent sound from being attentuated by radiating into the open air. But if the confinement were perfect, that is, if the walls of the room were completely reflecting, sounds would reverberate forever and get confused. To avoid these extremes, the walls in a real room must be partially absorbing. This has the effect of converting the discrete eigenvalues with perfectly reflecting walls into resonances. I will argue that for real rooms the width of resonances usually exceeds their spacing. This is important because it casts doubt on the usefulness of the concept of an individual mode in assessing the acoustic response of rooms; a smoothed description of the spectrum seems preferable. But smoothing is precisely what periodic orbit theory naturally describes. When there is no absorption, the contributions from the long periodic orbits make the convergence of the sum problematic, frustrating the direct calculation of individual eigenvalues, for example, in quantum chaology. Absorption attentuates the long orbits, and the oscillatory contributions from few shortest orbits are sufficient to describe the acoustic response. But these few orbits are important: the crudest smoothing, based simply on the average spectral density, obliterates all the spectral oscillations and fails to capture the characteristics of most real rooms.

To assess the significance of absorption, start from the Weyl counting formula for the number N of modes with frequencies less than f, for a room of volume L^3:



if the speed of sound is $c = 330\,\mathrm{ms}^{-1}$,

$$N = \frac{4\pi L^3 f^3}{3c^3}.$$

In the presence of absorption, modeled approximately by an exponential amplitude decay time T, that is, intensity $\sim \exp(-2t/T)$, the resonance width corresponds to a frequency broadening,

$$\Delta f = \frac{1}{2\pi T}.$$

Thus, incorporating the reverberation time T_{60}, corresponding to 60-dB intensity reduction, that is, $T = T_{60}/3\log_e 10$, the number ΔN of modes smoothed over by the broadening is

$$\Delta N = 6\log_e 10\frac{L^3 f^2}{c^3 T_{60}}.$$

For estimates, we can choose the frequency middle A ($f = 440\,\mathrm{Hz}$). Then, for a small auditorium with $L = 6\,\mathrm{m}$, and a reverberation time $T_{60} = 0.7\,\mathrm{s}$, $\Delta N \sim 23$, which is unexpectedly large for such a small room. For the Albert Hall in London, where the effective $L \sim 60\,\mathrm{m}$, and taking $T_{60} = 2\,\mathrm{s}$, $\Delta N \sim 8{,}200$. These estimates strongly suggest that there is little sense in studying individual modes.

Second, here is an unusual application of spectral statistics from 1993, inspired by a visit to Loughborough University, where I talked about quantum chaos and mentioned that the ideas could be usefully applied in acoustics. Afterward, Robert Perrin showed me his measurements (Perrin et al. 1983) of eigenfrequencies of one English church bell, ranging from 292.72 Hz – the lowest mode, called the hum, through the first few harmonics, with their traditional names Fundamental, Tierce, Quint, Nominal, Twister, Superquint – up to the 134th frequency of 9,285 Hz. This provided sufficient data to make a first attempt to understand the frequency spacings distribution.

I did this in two ways. First, taking the whole set of 134 frequencies, unfolding them by fitting the counting function (spectral staircase) to a cubic function, and then calculating the 133 spacings, normalized to unit mean. The resulting cumulative spacings distribution $C(S)$ = fraction of spacings less than S, fits the Poisson distribution $1 - \exp(-S)$ reasonably well (the thin and dashed curves in the figure). This is not surprising because the bell has approximate rotation symmetry, and the whole set of frequencies conflates subsets with different numbers l of nodal meridians ("angular momentum quantum number"). Fortunately the value of l for each frequency was given; l ranged from 0 to 28, but only the subsets with $0 \leq l \leq 10$ included sufficient frequencies to generate sensible statistics. In the second procedure, I unfolded these subsets separately and conflated the spacings afterwards, thereby generating the heavy curve in the figure. This is better fitted to the Wigner cumulative distribution $1 - \exp(-S^2/4)$ (the dotted curve in the figure), indicating strong repulsion of neighboring frequencies in each l-subset. The precise fit is not important because the Wigner distribution should apply when the ray geodesics on the bell – "classical paths" – are chaotic, whereas the vibrations of the bell, regarded as a thin elastic sheet, are probably integrable, with frequencies given by the modes of a one-dimensional "radial" equation, albeit of fourth order.

Reference

Perrin, R., Charnley, T. & DePont, L. (1983), "Normal modes of the modern English church bell," *J. Sound. Vib.* **90**, 29–49.