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Classical limits

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Like many other limiting connections between physical theories [1, 2], the classical limit is much more subtle than usually thought, even in ordinary nonrelativistic quantum mechanics. Two distinctions are not usually made but should be. The first is between the connections between the classical and quantum *equations*, and between classical and quantum *phenomena*, that is: *solutions* of the equations. The relation between equations was fairly well understood from the beginning: it involves Planck's constant, considered as a parameter that can be made formally small, leading to limiting connections between commutators and Poisson brackets, the Schrödinger equation and the Hamilton-Jacobi equation, etc. The relation between between classical and quantum *phenomena* is much richer and more subtle, and still being unravelled today, and I will concentrate on that.

Concerning the connection between classical and quantum phenomena, there is the second important distinction, between two fundamentally different questions [3]. The first is: *How does classical behaviour emerge from quantum behaviour?* This is essentially a reassurance exercise: we know in advance that in almost all cases where Planck's constant is small compared with relevant classical actions, quantum effects will disappear and the classical world *will* emerge – somehow. But it is a tricky reassurance exercise. We know now that it involves more than small-h asymptotics of solutions of the Schrödinger equation (WKB, etc). Also important is the fundamental fact that no system can be completely isolated from its environment: *decoherence*. The origins of decoherence can be traced back to Thomas Young [4], who had to think hard about why he found interference so difficult to see. Decoherence is central to understanding the classical limit because when h is small, quantum systems become exquisitely sensitive to uncontrolled environmental disturbances – interactions so weak that one would never think of including them in the hamiltonian. This has surprising consequences, arising from the mathematical fact that the classical limit and the long-time limit do not commute, and especially when there is chaos [3]. For example, the chaotic tumbling of Saturn's satellite Hyperion would, if isolated, reveal its quantization as a huge quantum rotator in only a few decades, because quantum mechanics suppresses classical chaos. But this suppression involves quantum coherence, which 'the patter of photons' from the sun will destroy in a time estimated at 10^{-50}s: decoherence suppresses the quantum suppression of chaos.

The second question is more positive, more creative, because it has led to the discovery of new phenomena: *In systems protected from decoherence, what quantum – that is, nonclassical – effects, emerge in the classical limit?* [3]. This sounds paradoxical, and there would be no such effects if it were not for another mathematical fact: for all except trivial cases, quantum solutions are *nonanalytic* functions of h at h=0 [1, 2]. Some phenomena in the quantum-classical borderland, associated with this nonanalyticity are: interference patterns near the singularities of families of classical paths – that is, near caustics [5, 6]: and, in quantum chaology, random-matrix distributions of high energy levels [7, 8], gaussian random morphologies of wavefunctions [9, 10], and nonclassical absorption of energy after sufficiently long times.

I have no space to go into details, but here is one pretty example of an extreme and semiclassically persisting quantum interference effect: *quantum revivals*. The early stages of an evolving quantum wavepacket representing a particle in a one-dimensional box are unsurprising (though a spacetime image of the evolution is striking [11]): as the classical particle bounces, its quantum counterpart interferes with its reflection; and it soon spreads throughout the box because of Heisenberg, since the packet contains a range of classical momenta. Less obvious is the fact that after a classically long time (of order mass×size^2/h) the coherence of the reflections leads to a quantum revival: the original packet reconstructs perfectly. At intermediate rational fractions p/q of this time, there is a superposition of q copies of the original packet, whose phases (and much more structure [12]) are determined by the gauss sums of number theory . This intricate 'quantum carpet' [11] has its counterpart in optics: the Talbot effect [13], observed in 1836 (near Bristol, as it happens) but understood only recently [14].

As well as the physical phenomena associated with nonanalyticity, explorations of the classsical limit have led to advances in mathematics: deeper understanding of divergent series [15-18], especially in situations previously regarded as not summable and in ways that lead to super-accurate numerics, and tantalizing progress towards the Riemann hypothesis of number theory [19, 20] which we now understand as intimately connected with quantum chaos [21, 22].

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