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Inauguration of Pippard's pendulum

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In Milton Keynes, the open university owns a deconsecrated church, sometimes used for meetings. Once, I stood in its pulpit and delivered a scientific sermon. That was strange, but even stranger was travelling the very next day to Bologna in Italy and finding that every lecture hall in the university had a crucifix above the door, to the acute embarrassment of the professors there, communists and atheists all. Well, I'm not religious, but it is a pleasure, and feels not at all strange, I discover, to speak in a real, that is, a working, church, to introduce the science of this hypnotic device, whose installation continues the tradition of putting works of art, and astonishing artefacts, in places of worship (the clock in the cathedral in Wells comes to mind).

What is it? Simple metaphors fail. It's not a pendulum, or a seesaw, or a pair of scales, although it does resemble those things. Brian Pippard's creation - Rob Knight's construction - is - well, a chaos machine.

In its original use, the word chaos was theological: it meant "without form, and void" as in the world before the creation as imagined in the bible, at the beginning of Genesis. And, faced with chaos as commonly understood today, referring to unpredictability or disorder, some people have turned to religion, and some (the same people, or different people) have turned to science, seeking principles to discover or impose order. There is some old science that attempts to come to terms with unpredictability, namely probability and statistics, which deal in precise ways with large numbers of events that are individually unpredictable. If I throw a die a million times, the fraction of fives (or ones, or sixes, etc) will be - precisely - close to one-sixth, even though it is humanly impossible to predict the outcome of any single throw.

How could the unpredictability of the world be understood, even though the laws of physics showed an iron determinism, where every future event will follow inevitably from how things are today (at least before the discovery of the quantum world, that plays no part in this story)? Most scientists would have said that events like coin tosses and throws of a die, and, even more, who we will meet and marry, and when and how we will meet our deaths, are unpredictable because they are the outcome of many different causes and we cannot know them all.

The new science of chaos comes from our recent discovery that this is only part of the story. There are some processes that are governed by very simple dynamics - only a single cause, if you like - and so they are determined by the simplest physics we know, but they are, nevertheless, as unpredictable as the national lottery. This seems strange, but on reflection it is deeply satisfying to find that such ancient opposites as randomness and determinism, chance and necessity, luck and fate, can in fact be intimately bound together.

With the water turned off, this heavy pendulum, with its light hollow cross-beam, would swing back and forth for a long time - forever, if the small amount of friction were eliminated - with perfect regularity. That is because at each swing gravity acts on the mass of the pendulum, to pull it back to vertical - and hence the cross-beam back to horizontal: the dynamics is *stable*. But when the water does flow, the swings become erratic. Why? Not because the stream of water, that drives the motion, is itself erratic: it is designed to be as smooth and even as possible. In fact the intrinsic dynamics is unstable, as I will explain.

Water pours in at the top and fills one of the arms of the cross-beam. Which arm? The one that slopes down. While the water is in the arm, its weight unbalances the see-saw, in the sense of giving it a turn tending to increase the downward slope. This is a *de*stablizing effect, opposite to that from the mass of the pendulum, which as just described acts to stabilize the motion and restore the cross-beam to horizontal. No sooner has water filled the arm, and destabilized the motion, than it empties out again, and the whole process repeats. Each emptying happens in a time short compared to the natural period of the pendulum when the water is not flowing, and the fill/empty cycles repeat at intervals unrelated to this period (and to do with how fast the water is flowing). These two contrary influences - the weight of the pendulum and the weight of the water in one arm of the cross-beam, conspire to generate chaos, that is motion which does not repeat, at least for a very long time.

One way to diagnose chaos would be to look at one of the arms (the right-hand one, say) at the instants when it is momentarily at rest as it reverses direction, and ask whether it is above or below the horizontal. Without the flow, the sequence is a simple alternation: above, below, above, below... With the flow, if the motion is chaotic, the sequence will be without pattern (e.g. above, above, below, above, below, below, below...). From part of it, it is impossible to predict the rest.

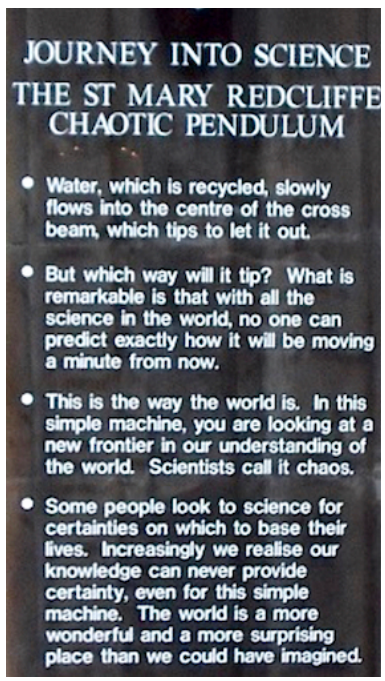
It is far from obvious why this should be so. One way to get understanding is by stripping the dynamics down to its essentials. I had fun with this. It is not necessary to simulate every detail of the device - the sloshing and dripping of the water, for example. The motion of the cross-beam is described by the angle made by one of the arms with the horizontal. At regular intervals, the rate of turning (angular velocity) gets a jump (representing the effect of water in one of the arms): positive if the angle is positive, and negative if the angle is negative. What could be simpler?.

To study the effect of this simple rule acting for a long time, we draw a graph where one axis is the angle and one is the rate of turning. In this representation, the ordinary regular swinging of the pendulum is a circle - when the angle is large, the rate is small, and vice versa. Including the jumps from the water means that only an arc of the circle gets completed; then the rate jumps - up or down depending if the angle is positive or negative - then another arc, then a jump. Arc, jump, arc, jump....even this gets a bit complicated, so we decide only to look at each moment just after a jump has occurred, that is, when an arm has just emptied, and plot a dot. Dot to dot - just as in the children's comic-books. This 'stroboscopic' way of picturing the motion is a simple way of seeing what happens over very long times. But still too complicated to guess, so... off we go, to the computer

Now comes the surprise. The pattern of dots depends with fantastic sensitivity on exactly how long the arc is between jumps (in the machine, this is related to how fast the water flows) and exactly how (at what angle and rate of turning) the pendulum is started off. Here is one dot pattern for 1000 (jumps) emptyings. After 10000 jumps, this is revealed as a tiny part of a much larger pattern. 30000 jumps, and more gets explored. My guess - and it is a guess, because (amazingly) it could be that the full consequences of even this simple rule (half a dozen lines of computer code) have not been fully investigated - is that eventually the dots would get further and further from the centre, corresponding to a very slowly growing instability.

This is without friction. Introduce a tiny amount - corresponding to a loss of one-thousandth of the pendulum's energy in each swing - and the pattern looks totally different. Eventually the dots get attracted to ten positions near the centre of the original pattern. This corresponds to the pendulum settling into a periodic motion that repeats after ten water-fillings. Not chaos, exactly, but hard to distinguish except by careful observation. Note that despite the friction, the motion never stops: the falling water supplies the energy needed to keep the pendulum going. And look at this. Reduce the friction to one *ten*-thousandth. The motion is completely different! Now it explores a bit more of the frictionless pattern, before settling into a period-three motion.

This clever machine, and this little geometrical abstraction of it, illustrate a deep principle, reversing the old idea that the world behaves in a complicated way because it involves many interacting parts, that is because it is made complicated. The principle is that simple rules can generate infinitely complicated behaviour. That's all.



Mathematical model

The model is that with A being the angle of the crossbar with the horizontal, and V the angular velocity, the pendulum rotates freely and is then, after turning freely for an angle α, given a unit angular impulse whose sign depends on the sign of A (that's the discontinuous part). I also incorporate a friction factor f (close to 1). So the map, starting from A_0, V_0, is

$$\begin{aligned} A_{n+1} &= f\left(A_n \cos\alpha + V_n \sin\alpha\right) \\ V_{n+1} &= f\left(-A_n \sin\alpha + V_n \cos\alpha + \text{sign}\left[A_{n+1}\right]\right) \end{aligned} \tag{1}$$

You get nice stroboscopic pictures (after each kick) in the *A*, *V* plane. In those below, I take $A_0=0$, $V_0=0.1$, $\alpha=\pi/\sqrt{3}$, 100000 iterations, and different frictions *f*: top left: $f=1$ (no friction); top right: $f=0.99$; bottom left: $f=0.999$, bottom right: $f=0.99999$